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Variability of observed drivers' car-following behavior on expressway basic segment

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Abstract

The inter-driver variability of follow-the-leader behavior is well recognized as a main cause of bottleneck phenomenon on expressway basic segment. The bottleneck phenomenon observed at sag sections on Japanese expressways is an example where the accumulation of driving behavior variability results in obvious capacity drop and probabilistically occurring breakdown. Conventional theoretic analysis on drivers' car-following behavior generally treats the variability as a random error around a standard behavior pattern, which is not sufficient to study the exact pattern and extent of the variability to provide any acceptable explanation to the probabilistic occurrence of breakdown as well as applicable solution to unify the behavior. Trajectory data from video processing provides a source for this research to analyze every single driver's follow-the-leader behavior pattern and extent of variability from a microscopic view as well as their impact on overall traffic flow condition.
875 time series profile of single drivers' behavior such as acceleration, speed, distance and road geometry is studied. The follow-the-leader behavior of every driver is fitted into a car-following model, where drivers decide their acceleration based on the speed difference, spacing with its leading vehicle, its own reaction delay time, target spacing as well as the road geometry.
Reaction delay time and desired spacing are obtained for every driver through correlation analysis and regression. Other parameters are obtained with heuristic optimization algorithm. The car-following model is calibrated with objective function and performance indicators as well as simulation. The objective function and performance indicators are studied to ensure that the model and its parameters can reproduce car-following behavior well. The simulation is carried out to reproduce the probabilistic nature of traffic congestion occurrence at a bottleneck of sag section including the effect of vertical slope increase. It successfully reproduces the tendency that the congestion occurrence probability grows with traffic demand. Finally, the variability of car-following behavior in drivers at sag

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section is studied with car-following parameter distribution. The desired spacing parameters are jointly distributed. The vertical effect parameter has two peaks concentrating on the value range, which indicates that there are drivers who are not affected by the vertical grade while others are.

Keywords: car-following behavior; expressway basic segment bottleneck; variability in car-following behavior;

1. Literature review

Sag sections are the major basic segment bottlenecks on Japanese expressways. A sag section is a road segment on an expressway in which the vertical slope increases at a small but constant rate. Because the rate of slope increase is slight and constant, vehicle speeds are significantly reduced before drivers become aware of the change in slope. According to a research report by the Japanese National Institute for Land and Infrastructure Management, 60.5% of congestion incidents on Japanese expressways in 2011 occurred on uphill slope and sag sections and caused congestion amount of 105,649 km•h(National Institute for Land and Infrastructure Management).

Traffic managers find an urgent need to monitor and predict the occurrence of congestion, which requires systematic observation on the bottleneck phenomena at sag sections and analysis of its mechanism. The first systematic observation was made by Koshi et., al (Koshi, 1986). Major features of traffic flow condition at sag section bottleneck were concluded. The bottleneck capacity would drop to around 2,200~2,700 veh/h/2lane, much lower than the theoretical capacity when the traffic demand reached to around 3,000 veh/h/2lane. Rapid speed reduction was observed at the start of bottleneck activation. On the other hand, as observed at many expressway bottlenecks, traffic capacity is a time-dependent feature and its definition remains not fully solved.(Banks, 1990) Congestion also occurs at a lower flow rate after a constant high flow passing the bottleneck. (Koshi, Kuwahara, & Akahane, 1992)(Patire & Cassidy, 2011) In other words, congestion occurs at same expressway bottlenecks under different flow rate.

After observing traffic flow condition at sag section for several years, researchers realize that the congestion is formed with an aggregation of individual car-following behavior. Xing and Koshi (Xing & Koshi, 1996) studied the car-following behavior of drivers at the start of congestion with aerial image data. They modeled car-following behavior for every individual driver with trajectory data. They found that 75% of drivers in a platoon decelerate intensely while their spacing with the leading vehicle decreases slightly. This finding suggests certain kind of car-following behavior maybe a possible explanation for congestion occurrence at sag sections. It also suggests that variability in the behavior of drivers may be the main cause of the probabilistic occurrence of congestion and the lack of a certain observable bottleneck capacity.

A simple understanding of congestion formation because of car-following behavior at sag section is: Cars first decelerate because of the vertical grade. When drivers noted the speed drop, they try to restore the speed by acceleration. A series of small adjustment follows after the unnoticed deceleration and intended acceleration. Drivers naturally tend to keep a longer spacing with leading car in order to have enough spacing for speed adjustment. Bottleneck is activated after the accumulation of speed adjustment of following drivers in the platoon passing the sag section and causes congestion occurrence.

The inter-driver variability is the difference of car-following behavior between different drivers. It not only reflects the variability of drivers' reaction to driving condition change of leading vehicle, e.g. acceleration and deceleration, but also reflects the difference of drivers reaction to vertical slope change. Some drivers are not affected by the unnoticed deceleration caused by vertical slope change as much as other drivers. By analyzing the inter-driver variability of car-following behavior, it is possible to reveal the mechanism of the probabilistic congestion occurrence at sag section.

Car-following behavior of every single driver during congestion formation with observation data from a famous sag section in central Japan is modeled in this research. A car-following model considering the vertical grade effect is utilized and calibrated for this purpose. Distributions of parameter values reflect the variability in car-following behavior at sag section.

2. Data

Two datasets were obtained through two observations carried out at the outer lane on a two lane basic segment on Tomei expressway in Japan. The Tomei Expressway connects Tokyo and Nagoya, which are the two major metropolitan areas in Japan. Connecting with Meishin Expressway, it is on the most important artery with the heaviest traffic volume in Japan. A total of 875 cars were observed. Details of the datasets are given in Table 1.

Table 1. Detail of observation

Observation date	Time period	Congestion duration	Number of cars observed
Dataset 1			
Jul.15, 2006	4:30–7:30	6:15–18:50	104
Jul.22, 2006	4:30–7:30	6:45–11:10	92
Jul.29, 2006	4:30–7:30	5:55–13:40	78
Aug.4, 2006	6:00–9:00	7:25–12:55	119
Total			393
Dataset 2			
Nov. 2010~ Aug. 2011	-	-	482

Dataset 1 is a trajectory dataset obtained by Muto and Akahane(Muto & Akahane) from a 1.2-kmsection of roadway, including a sag section called Yamato sag, on the Tomei Expressway during congestion formation. The trajectories were processed through several fixed-site videos along the observation area using video recognition technology. Position, speed, and acceleration data were obtained from the trajectories for all vehicles within the observation area, as illustrated in Fig. 1,every 1/30sand were properly smoothed for analysis. A total of 393 trajectories were observed. System errors occur in the video recognition, capture, and data smoothing processes. In addition, human driving behavior exhibits randomness. This is especially noticeable when there is no specific stimulus to drivers to change their driving status. Therefore, a study area for follow-the-leader behavior modeling was further extracted from the overall trajectory to focus on car-following behavior at a sag section as well as minimizing the errors that occurred in the video recognition, capture, and data smoothing processes. The study area in the dataset is 500~650m area from 21.85 km to 22.35~22.5 km (See Fig. 1). The starting point is chosen to avoid errors occurring in video recognition and smoothing process. The end point is chosen to avoid lane-changing and other behaviors that may affect the analysis.

Dataset 2 is a dataset of 482 trajectories during congestion formation at the same location obtained by NILIM. The observation was carried out on weekends during Nov. 2010 to Aug. 2011. The data consisted of distance for every 0.1s and speed and acceleration for every 1s. The speed was filled with linear interpolation for every 0.1s.

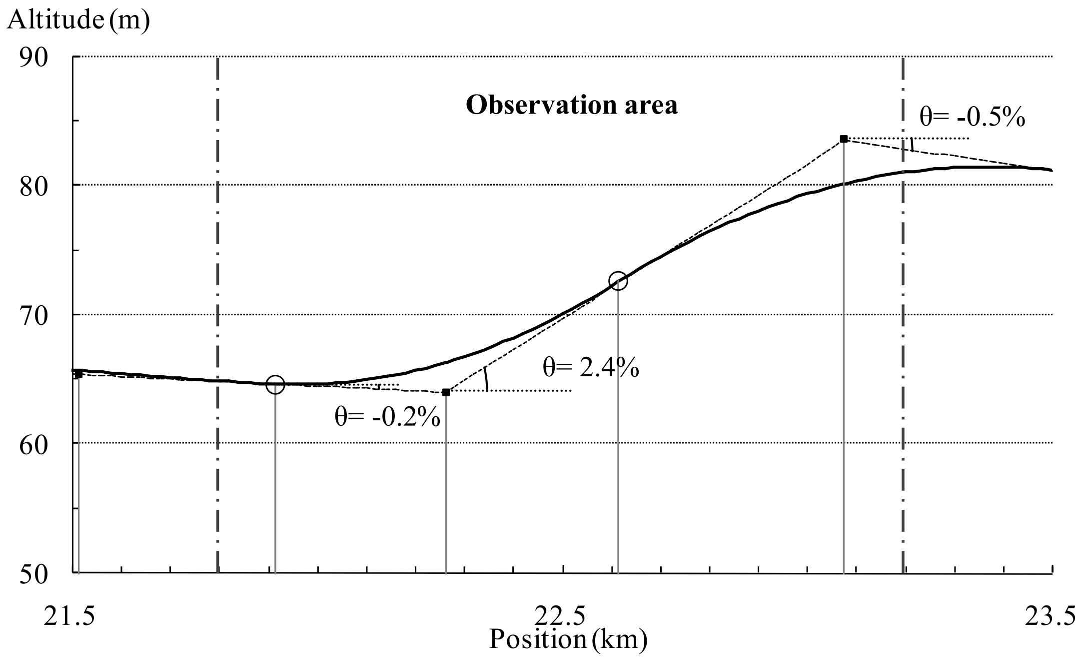


Fig. 1. Observation location

There were a few vehicles changing lane to overpass the preceding vehicle during observation. The spacing of vehicle decreases severely while speed increases when vehicle change lane to overpass the preceding vehicle. The lane-changing behaviors are thus recognized from dataset and are excluded from analysis as error data. Two kinds of vehicles that are not under car-following state are also excluded from analysis. One is vehicles under independent driving state. Vehicles under independent driving state are far enough from their preceding vehicles that their driving behavior is not affected by the behavior of preceding vehicles. They are independent in driving state instead of under car-following state. Time headway and spacing are common criterion for estimating whether vehicle is under car-following state or not. Highway Capacity Manual suggests that vehicles travelling with a headway smaller than 3s is considered as following vehicles(TRB, 2000). Pasanen and Salmivaara(Pasanen & Salmivaara, 1993) also suggested 3s be the threshold for following vehicles and non following vehicles. Vogel (Vogel, 2002) found that the time headway for urban roads is around 6s and the spacing is 50-70m. Both criterion are used in this study. Vehicles having spacing bigger than 80m or time headway bigger than 3s with preceding vehicle are regarded as vehicles driving independently regardless of preceding vehicles' driving state and eliminated from car-following behavior analysis.

The other is vehicles already involved in congestion. Vehicles already involved in congestion are forced to maintain a low speed equal to it preceding vehicle and a close spacing. Their behavior is highly restricted within the dense low speed flow so that it is not a normal car-following behavior. After congestion formed, the speed inside platoon is small and the density of traffic flow is high. The traffic flow inside platoon is condensed that the speed of leading and following vehicle will be forced to decrease to the speed inside the congestion. Therefore, instead of following the leading vehicle and pursuing desired spacing, the following vehicle is simply force to maintain the same congestion speed as the leading vehicle in the congestion. Considering the research objective is the car-following behavior of vehicles during free flow to the formation of congestion, this kind of vehicles involved in congestion is eliminated from analysis. Congestion formation at sag section is a dynamic process. There are many definitions of congestion occurrence (Oguchi, Katakura, & Shikada, 2001)(Xing & Koshi, 1996; Xing, Tsuru, Ishida, & Muramatsu, 2010). A simple and practical definition of congestion occurrence at sag section is two adjacent vehicles speed smaller than 40km/h at the same time. This threshold is used in this research. As long as there are two adjacent vehicles' speed smaller than 40km/h, the later vehicles are regarded as involved in the congestion and eliminated from car-following behavior analysis. The summary of observation data and effective samples are shown in Table 2.

Table 2. Detail of observation

	all	error	independent	congested	effective samples
Dataset 1	393	64	7	40	**217**
Dataset 2	482	27	63	0	**326**

3. Methodology

In this research, A model first proposed by Oguchi and Konuma(Oguchi & Konuma, 2009) that has a vertical force effect component added to Helly's model as shown in function (1) is used.

$$a_1(t+\Delta t)=\begin{cases}\alpha_1(v_0(t)-v_1(t))+\alpha_2(x_0(t)-x_1(t)-s_1^*) & t<t_0\\ \alpha_1(v_0(t)-v_1(t))+\alpha_2(x_0(t)-x_1(t)-s_1^*) & \\ \qquad -\beta g[\sin\theta(t)-\sin\theta_u] & t>t_0\end{cases} \quad (1)$$

$$s_1^*=\delta+\tau v_1(t)$$

where,

- Δt = Driver's reaction time
- $a_1(t)$ = Acceleration of following vehicle
- $v_0(t)$ = Speed of leading vehicle
- $v_1(t)$ = Speed of following vehicle
- $x_0(t)$ = Position of leading vehicle
- $x_1(t)$ = Position of following vehicle
- s_1*= Desired spacing of following vehicle
- g = Gravity acceleration
- $\theta(t)$ = Vertical slope
- θ_u = Initial vertical slope before sag section
- α_1, α_2, β = Coefficients
- δ = Standstill spacing
- τ = Spacing change with speed change
- t_0 =Time of entering the sag section

There are two reasons for using Helly's model as the basic model. One is that the physical relationships of acceleration, speed difference and spacing represent the actual experience in observation well. As Helly pointed out, in a platoon with comparatively high density, vehicles only try to keep up with the leading vehicles. Therefore, they mainly react to the difference in the state with its leading vehicle. In driving experiments at sag sections, Helly's model appeared to fit to the car-following behavior well (Shibuya, Oguchi, & Hong 2013)(Oguchi & Konuma, 2009)Helly's model can also reproduce the overreaction of drivers in speed reduction observed by Yoshida(Yoshida, Koshi, & Yasui, 1997), while some models with more complex structure like Intelligent Driver Model appear to produce impassive reactions. The other reason is that a simple linear model with limited numbers of variables will make estimation and analysis of variables easier as well as manifest clear physical relationships in parameters distribution and simulation.

The parameters of driver showed significant difference at different vertical gradient sections. Oguchi and Konuma (Oguchi & Konuma, 2009) used the Helly's model among other classic car-following models to analyze the car-following behavior at sag sections in Japan with experiment data. Helly's model showed good fitness in these researches. Sag effect parameter β reflects the unnoticed deceleration caused by the vertical change. The inter-driver variations are described by the different values of parameters of different drivers.

Desired spacing can be described with more complex structures, such as a linear function of spacing, speed and acceleration(Helly, 1959) as well as curves with different bending direction for free flow and congested flow (Xing & Koshi, 1996). In this research, desired spacing is assumed to be a linear function of speed because the objective is to model the formation of congestion where the speed of vehicle is between 40km/h to 100km/h, which is at the flat

part of speed and spacing curve in speed-spacing curve. Acceleration only has a minor effect(Helly, 1959) on desired spacing and thus ignored.

The desired spacing and reaction time range can be directly obtained from observed data. On the other hand, $\alpha 1$, $\alpha 2$, β and the exact value of Δt is unobservable and therefore obtained with cross-entropy method. The ranges of the parameter values were set as follows: $\Delta t \in[0,5]$ s, $\alpha 1 \in [0,1]$, $\alpha 2 \in [0,0.3]$, $\beta \in [0,1]$, $\delta \in [0,100]$, and $\tau \in [0,5]$.

The analysis of driving behavior variability is carried out with 3 steps.

1) The desired speed s_1* is estimated with regression of vehicle's speed spacing relationships under steady driving state (a<0.01g).

2) The range of reaction time Δt is estimated with correlation analysis within 0~5 seconds.

3) The other parameters are estimated with cross-entropy method, where objective function is the relative percentage error of spacing and 3 performance indicators are also studied.

3.1. Desired spacing

Vehicles always adjust the spacing and speed difference with preceding vehicles. This dynamic adjustment is shown as accelerate and decelerate in driving states. When the speed of vehicle is bigger, it tends to keep bigger spacing with preceding vehicle. When the speed of vehicle is smaller, it keeps smaller spacing with preceding vehicle. There exists a desired spacing for every vehicle to maintain with its preceding vehicle under certain speed. When vehicle achieves its desired spacing, it will maintain a certain speed if there is no further change of traffic condition. Steady driving state is the state when vehicles maintain a very small acceleration or deceleration. At steady driving state, vehicles don't accelerate or decelerate to adjust to the driving state of preceding vehicle. Therefore, the steady driving state reflects a state of vehicles under some ideal speed and spacing relationship. Steady driving state is defined as acceleration smaller than 0.01g according to previous research experiences (Oguchi & Konuma, 2009)(Shibuya, Oguchi, & Hong 2013). A simple way of reflecting this relationship between desired spacing and speed is linear relationship(Newell & Frank, 2002).

For the steady driving states of all the trajectories the value of $\delta 0$ = 5 and $\tau 0$= 1.2078 are estimated with linear regression (r2=0.2176) as shown in Fig. 2. The free independent driving state of headway bigger than 3s is excluded from estimation.

Therefore, the desired spacing of all the vehicles are as shown in function (2).

$$s_1^* = 5 + 1.2078 \cdot v_1(t) \tag{2}$$

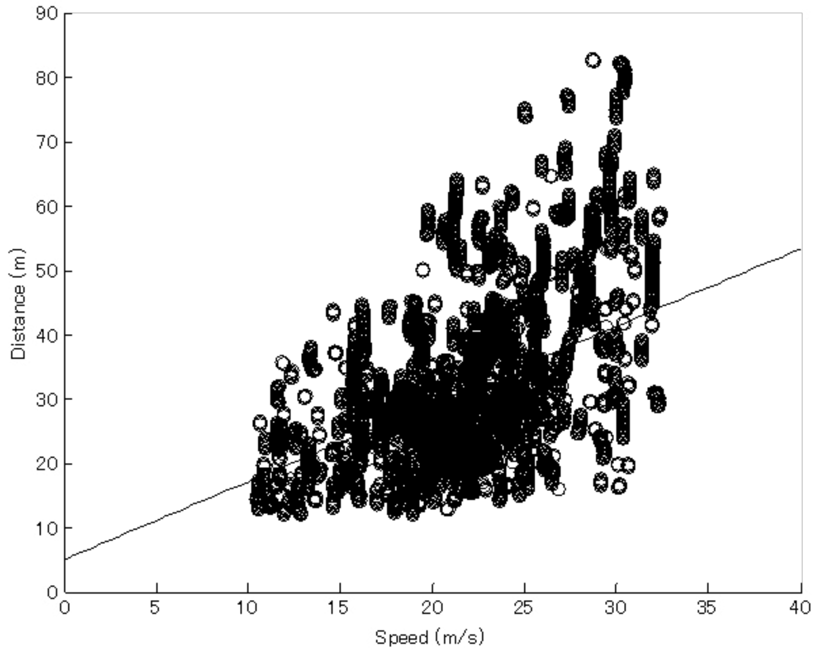


Fig. 2. Desired spacing estimation with all vehicle trajectories

It can be observed from Fig. 2 that the desired spacing of drivers under high speed (>25 m/s) is more dispersed than that of low speed.

The speed-desired spacing relationship is also estimated for every separate vehicle. There are two kinds of condition in observed speed and desired spacing relationship.

One is that several steady driving states exist in a trajectory, δ and τ are estimated from the steady driving states. The data is a time-series data, thus the errors also have a time-series structure which may be accumulated to cause bias in regression. To avoid the bias caused by errors, Tobit model is used instead of linear regression. The Tobit model supposes that the dependent variable is latent, which is linearly dependent on independent variable and consists an additional normally distributed error term (See function (3)).

$$y_i^* = \beta \cdot x_i + u_i$$
$$u_i \sim N(0, \sigma^2) \quad (3)$$

The value of τ is set to $\tau \in [0, 5]$. The desired spacing should not be smaller than a minimum distance when speed is low. According to a research of the U.K. Department of Transport(Department for Transport and Driver and Vehicle Standards Agency, 2007), it is generally established that the stopping distance of vehicles at 20mph (32km/h) is 12m. The spacing is 16m considering the 4m length of vehicle. Therefore, the estimated desired spacing should not be smaller than 16m when the speed is at 32km/h. The lower bound of dependent variable is 16m. δ and τ are estimated with maximum likelihood in Tobit model.

The result is shown as Fig. 3. δ is the intercept and τ is the slope of the estimated line.

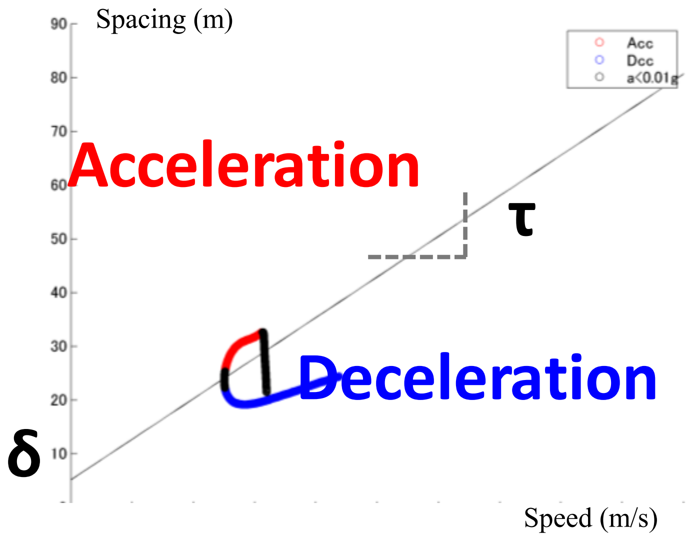


Fig. 3. Desired spacing estimation with individual trajectory (several stable states)

The red dots represent the acceleration phase of vehicle and the blue dots represent the deceleration phase of vehicle. Vehicle accelerates when the spacing is bigger than its desired spacing and decelerates when the spacing is smaller. These two different phases are well separated by the estimated desired spacing line in Fig. 3.

In the other condition, there is only one steady driving state in a trajectory, where the speed difference of steady driving state is smaller than 5%. This condition occurs especially when obvious propagation of deceleration is observed. Vehicle has less time to adjust to their desired spacing than to decelerate like the leading vehicle. Under this condition, the desired spacing is assumed to be a constant where τ=0, and δ is the mean speed of the steady state in the desired spacing function.

To conclude, the approaches taken in desired spacing estimation are:

(1) Several steady states exist, desired spacing function parameters are estimated by Tobit model with maximum likelihood estimator. $\tau \in [0, 5]$ and $s_1^* \mid v_1(t)$=40 km/h >16m.

(2) Only one steady state exists, τ=0, and δ is the mean speed of the steady state.

(3) No steady state, use the desired spacing parameters for all the trajectories: $s_1^* = 5 + 1.2078 \cdot v_1(t)$

3.2. Reaction time

The parameter Δt reflects the reaction time of a driver to the relative speed and spacing difference from the desired spacing with the leading vehicle, as expressed by function (7). The most suitable value of Δt should generate the best correlation of the driver's acceleration with either the relative speed or the spacing difference from the desired spacing with the leading vehicle. Therefore, Δt was estimated from the results of a correlation analysis within the range of [0,5] s. The analysis area was extracted to capture the dynamics of speed change (See Fig. 4).

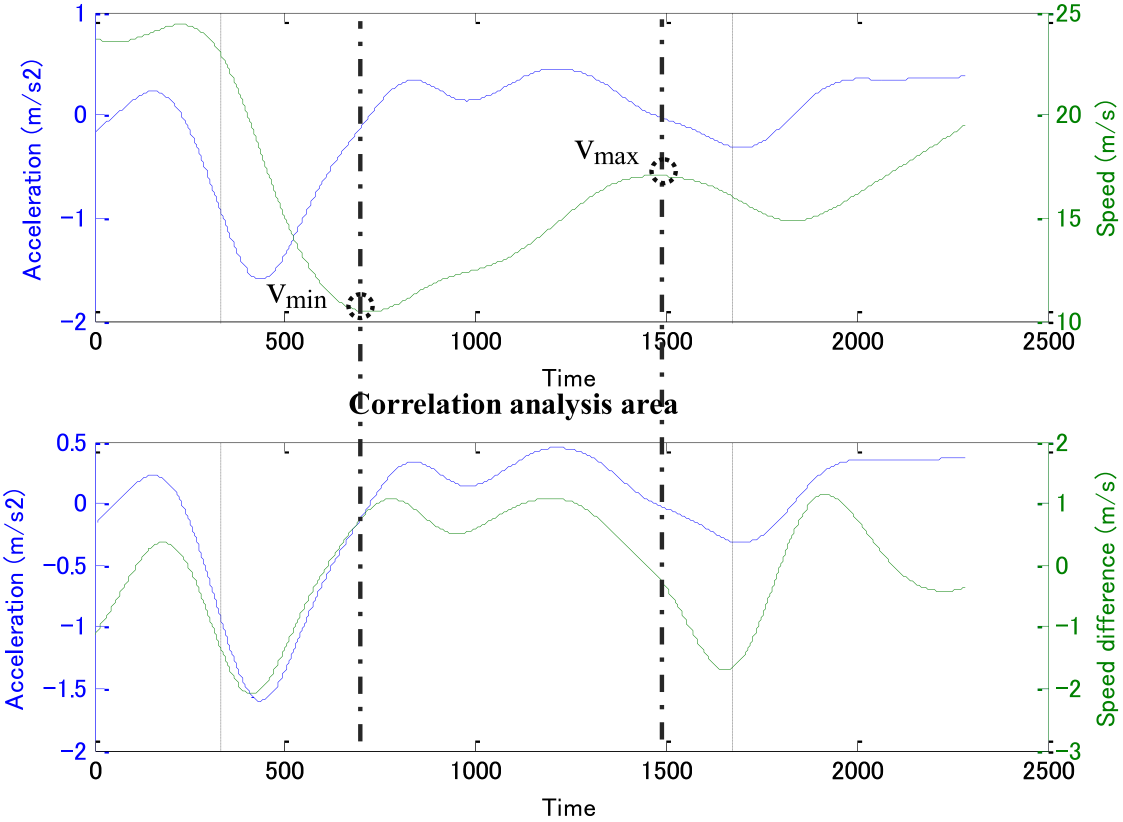


Fig. 4 Correlation analysis area of Δt

The correlation analysis of reaction time is carried out on trajectories with enough change in driving state in terms of difference of maximum speed and minimum speed bigger than 10% as shown in function. When speed change is small, the acceleration and deceleration are more due to other random adjustment instead of car-following incentive to adjust to the driving state of preceding vehicle.

Correlation distribution of acceleration with relative speed and spacing difference with desired spacing with reaction time of 0~5 seconds are studies and the reaction time range of Pearman's correlation r>0.8 were extracted (See Fig. 5, where the correlation of acceleration and relative speed is shown in blue line according to the left axis and the correlation of acceleration and spacing difference is shown in green line according to the right axis. The extracted range of reaction time is within the dotted lines). Of the best correlations between acceleration and relative speed more than 50% are greater than 0.8, which indicates a strong correlation between acceleration and relative speed.

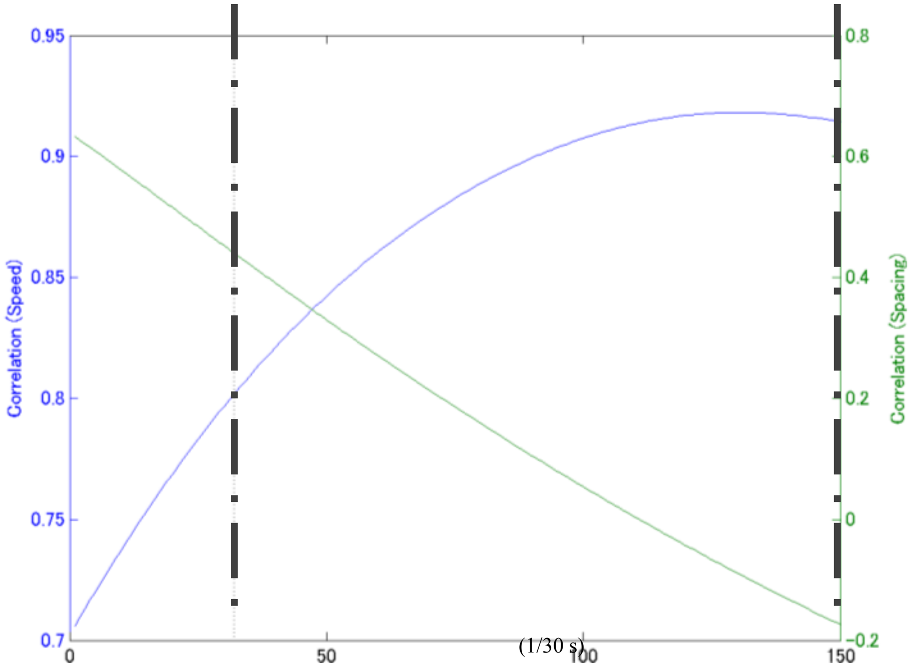


Fig. 5 Reaction time range with correlation analysis

It can be observed that the correlation of acceleration and relative speed as well as the spacing difference changes gradually with reaction time. Therefore, there exists a wide range of high correlation of acceleration and relative speed /spacing difference. Meanwhile, the changing direction of correlation with relative speed is different from the correlation with spacing difference in the figure. The correlation with relative speed increases when reaction time is bigger, while the correlation with spacing difference decreases when reaction time is bigger. Considering of the structure of car-following model where acceleration is affected by both the relative speed and the spacing difference, both should be considered in the calculation of reaction time. Helly suggested that relative speed had a major influence on acceleration within the modeling structure. So the correlation of acceleration and relative speed is given priority in searching reaction time. The reaction time is thus searched within the range of correlation of acceleration with relative speed bigger than 0.8 so that both relative speed and spacing difference is considered. If both best correlations were less than 0.4, which would indicate no significant correlation in the observed trajectories, Δt is to be searched with cross-entropy method within $\Delta t \in [0,5]$ s.

To conclude, the conditions and approaches in reaction time are:

1. If the maximum correlation of acceleration with relative speed>0.8, the range of reaction time with correlation of acceleration with relative speed>0.8 is taken as the range of Δt.

2. If the maximum correlation of acceleration with relative speed<0.8, and the maximum correlation of acceleration with spacing difference >0.8, t he range of reaction time with correlation of acceleration with spacing difference>0.8 is taken as the range of Δt.

3. If both the maximum correlation of acceleration with relative speed and spacing difference <0.8, the reaction time is to be searched by cross-entropy method within $\Delta t \in [0,5]$ s.

3.3. Heuristic search

It is difficult to obtain values for $\alpha 1$, $\alpha 2$, and β directly from observations, while it is easy to obtain reasonable values for δ, and τ from analysis of the trajectory data as shown in 4.1. For Δt, it is easy to estimate the range of value from observation data as shown in 4.2. But because Δt affected on both relative speed and spacing difference, so the

exact value of *Δ*t is hard to be estimated directly from observation. Therefore, four out of six parameters in car-following model need to be searched at the same time.

A heuristic search method called cross-entropy method is used to search for the best combination of $\alpha 1$, $\alpha 2$, Δt, and β fitted to the trajectory. It is based on Kullback-Leiber cross-entropy, importance sampling, Markov chain and Boltzmann distribution. The core concept in this method is to adaptively adjust the occurrence of the events more likely in the vicinity of a global extremum by using important sampling.

Mixed error measure of the relative error and the absolute error is used as objective function to balance the weight of distance in error.

The objective function is defined as function (4), which was proposed by Kesting et,al.(Kesting & Treiber, Calibrating Car-Following Models using Trajectory Data: Methodological Study, 2008).

$$\sqrt{\frac{1}{E(|s_{obs}(t)|)} \cdot E(\frac{(s_{obs}(t) - s_{sim}(t))^2}{|s_{obs}(t)|})}$$

(4)

where,

- sobs(t) – Observed distance;
- ssim(t) – Simulated distance;

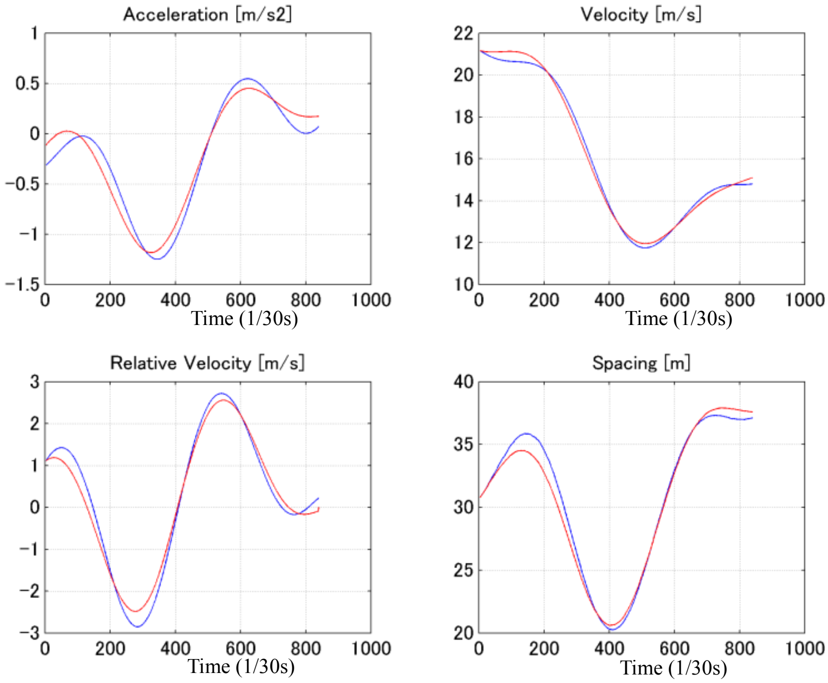
The root mean square error (RMSE) of the spacing, speed, and acceleration were used as performance indicators (PI) to reflect whether or not the actual driving conditions were properly described by the estimated parameter values(see function (5)). The vehicle behavior entering sag section are time series with a natural temporal order. So the dynamic change of car-following behavior is also very important for congestion formation. This dynamic in driving behavior is reflected by PI2 and PI3.

$$PI1 = \sqrt{\frac{1}{n}\sum (s - s_{sim})^2}$$
$$PI2 = \sqrt{\frac{1}{n}\sum (v - v_{sim})^2} \quad (5)$$
$$PI3 = \sqrt{\frac{1}{n}\sum (a - a_{sim})^2}$$

Cross-Entropy method was used to estimate parameters in this research. The core concept is to adaptively adjust the occurrence of the events more likely in the vicinity of a global extremum by using important sampling (Rubinstein, 1999). The weight of parameter values is equal. 5000 combinations of parameter values are generated at every round. The objective function is then calculated for every parameter value combinations. The parameter values are re-distributed within the range of the best 10% objective function values. The iteration continues until the best fitted values of parameters are finally estimated when the best objective function converge to a best objective function value.

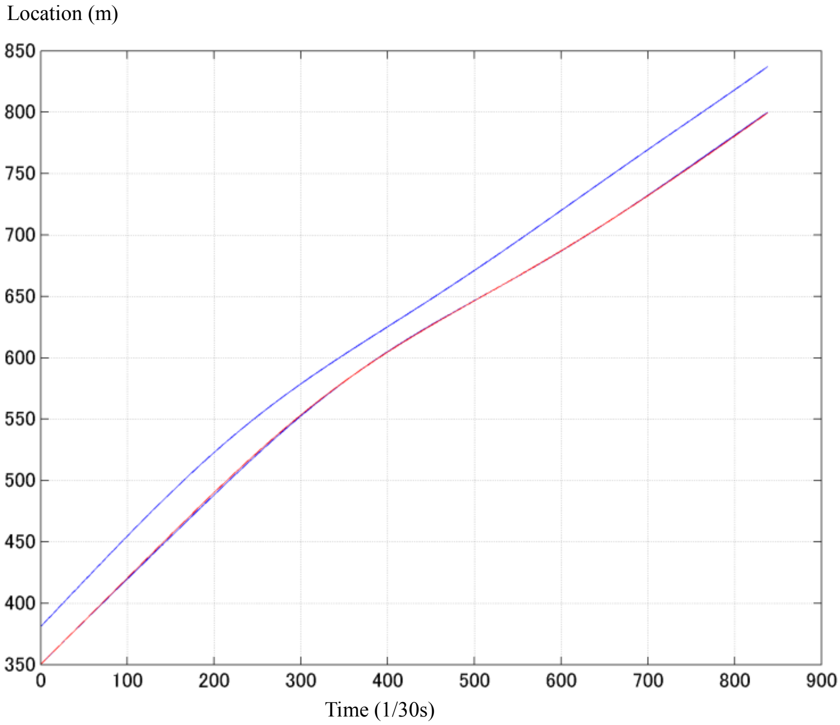
The search range of $\alpha 1$, $\alpha 2$ is estimated with several times of heuristic search. Initially a wide range of $\alpha 1$ and $\alpha 2$ are set: $\alpha_1 \in [0,1.5]$, $\alpha_2 \in [0,1]$, most estimated $\alpha 1$ and $\alpha 2$ are found to be within in the range of $\alpha 1 \in [0,1]$, $\alpha 2 \in [0,0.3]$ while a few cases are at both boundary. The cases with either $\alpha 1$ or $\alpha 2$ value at the boundary is a failure of the heuristic search because no extremum value is reached inside the range. In order to eliminate the failure of heuristic search and obtain reasonable parameter estimation values, the range of $\alpha 1$ and $\alpha 2$for heuristic search are finally shorten to $\alpha_1 \in [0,1]$, $\alpha_2 \in [0,0.3]$.

The fitted behavior of vehicle is shown in Fig.6 and The fitted trajectory is shown in Fig.7. The behavior of vehicle consists with acceleration, speed, relative speed and spacing with leading vehicle where the blue line stands for the actual behavior from observation and the red line stands for the fitted behavior with car-following model. We can see good fitness from both figures. The objective function and PI values are collected for further analysis.



Blue line: Actual data; Red line: Estimated

Fig. 6 The fitted behavior of vehicle with car-following model



Blue line: Actual data; Red line: Estimated

Fig. 7 The trajectory with car-following model

4. Calibration

Two methods are used for calibrating the car-following model. One is studying the objective function and performance indicators for all the drivers in order to exam how well the car-following model can reproduce the microscopic car-following behavior of drivers. The other is simulating the 5-min traffic condition with car-following parameters to exam how well the car-following model can reproduce the macroscopic traffic condition at sag section.

4.1 Objective function and performance indicators

The performance of objective function and PIs of both datasets are studied in this section. This performance indicates how well the car-following model and its parameters can reflect the actual condition.

The performance of car-following model in dataset 1 is firstly studied. Fig. 8 shows the PI1 (RMSE of spacing) in ascending order and the corresponding objective function values. With the same percentage of change in spacing, PI1 is bigger when the initial spacing is bigger. It can be observed that PI1 grows drastically when it's bigger than 4 and the corresponding objective functions are obviously big for small spacing cases. The physical meaning of PI1 is the average absolute difference of spacing in every estimation time step. The cases with PI1=4 means that at every time step, the average absolute difference of spacing is around 4m, which is about 10% of the average actual spacing. Therefore, PI1<4 is acceptable as the threshold for goodness of fit. The cases with PI1>4 and objective function >0.02 are obviously less well fitted than other trajectories. Based on this observation, the cases with PI1>4 and objective function >0.02 are excluded from analysis.

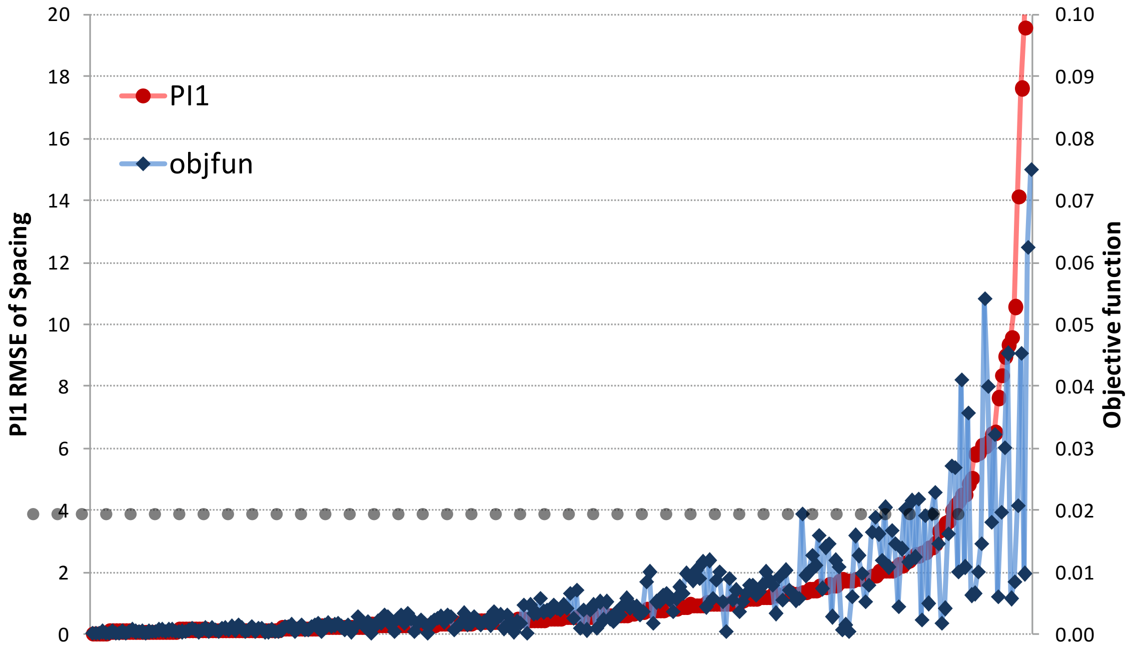


Fig. 8 Objective function and PI1 of dataset 1

The behavior of vehicles are further studied with PI2 (RMSE of speed) and PI3 (RMSE of acceleration) as shown in Fig. 9 in the same ascendant order of PI1. PI2 and PI3 The same drastic growth of PI2 and PI3 can be observed for cases with PI2>1 or PI3>0.5. A big PI2 or PI3 indicates that the dynamic behavior of driving behavior in term of speed and acceleration is not well fitted with car-following model. The trajectories cannot well reflect the dynamic behavior should be excluded from analysis since the car-following behavior of vehicle entering sag-section is a time series data. Therefore, the trajectories with PI2>1 or PI3 >0.5 are excluded from analysis.

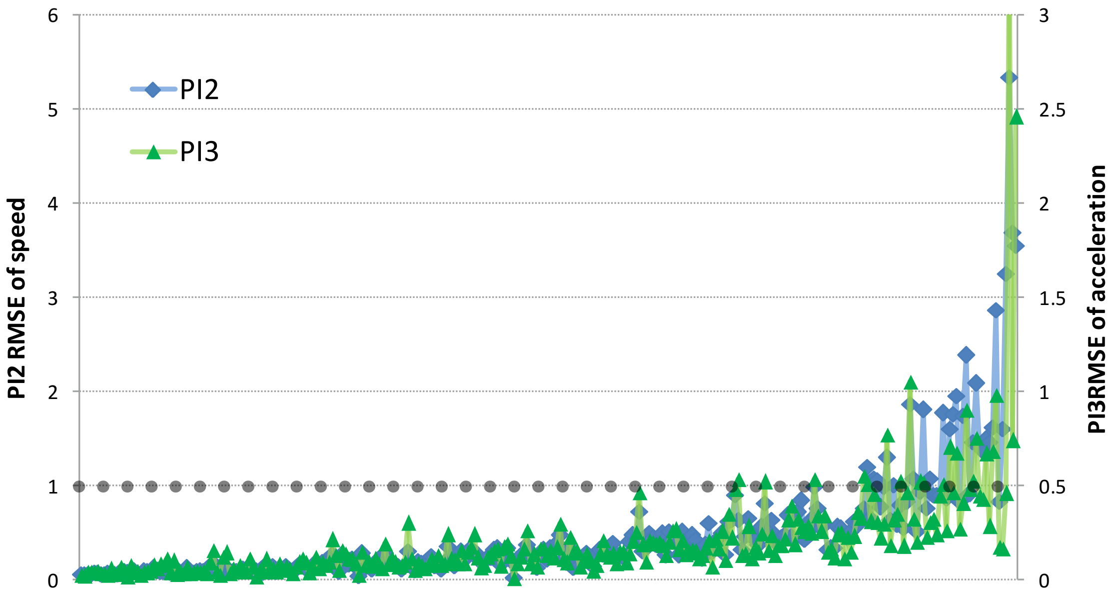


Fig. 9 PI2 and PI3 of dataset 1

The same analysis is also carried out for dataset 2. The spacing is obtained for every 0.1s in dataset 2 and the observation area length is around 1km, which is about two times the length of observation in dataset 1. The same tendency of value change in objective function and PIs can be observed in both figures. Therefore, the threshold of goodness of fit is set as twice the threshold in dataset 1 considering the study area length of dataset 2. The cases with PI1>8 and objective function>0.08 or PI2>2 or PI3>1 are excluded from analysis.

The final result of effective samples in field observation data is shown in Table 3.

Table 3 Summary of field observation data

	all	error	independent	congested	Not well estimated	effective samples
Dataset 1	393	64	7	40	65	**217**
Dataset 2	482	27	43	0	86	**326**

4.2 Simulation

A microscopic traffic simulation is made to simulate the car-following behavior at a single lane basic segment. Car-following model and parameter distribution are the results shown in previous chapters.

The vertical layout of the road is designed as the same as the observation site on Tomei expressway.

The initial speed is 28m/s (100.8 km/h) based on the average speed of vehicles before bottleneck. Vehicles entering the road segment randomly with a headway of negative exponential distribution. The criterion for congestion is two adjacent vehicles' speed smaller than 40km/h as presented in 3.3. The independent vehicles are those with spacing bigger than 80m or headway bigger than 3s.

The behavior of leading vehicle has an effect on every following vehicle in the platoon. It is generally recognized that speed perturbation of leading vehicle amplifies to following vehicles in a platoon. It is the amplification of this speed perturbation having an obvious affect on flow condition of the platoon. Therefore, three kind of leading vehicle's behavior were designed as function to reflect the actual deceleration of platoon leading vehicle and describe the speed perturbation at sag sections based on the observed behavior analysis.

Type1. The constant sag affected driver:

The constant sag affected driver decelerates when entering into sag section and keep the constant deceleration in the sag section. The function describing the constant sag affected driver is presented as following.

Type 2. The non speed restore driver:

The non restore driver decelerates when entering into sag section and realizing the speed drop after the reaction time. The non restore driver accelerates to balance the sag effect but not trying to restore its initial speed after realizing the sag effect. The function describing the non speed restore driver is presented as following.

Type 3. The speed restore driver:

The speed restore driver decelerates when entering into sag section and realizing the speed drop after the reaction time. The speed restore driver tries to accelerate to restore its initial speed after realizing the sag effect. The function describing the speed restore driver is presented as following. The acceleration part to restore initial speed is proposed by Newell et.,al.(Newell & Frank, 2002)

Vehicles are design to enter simulation in equilibrium travelling state. In the car-following model, vehicles react to the speed difference and space headway with leading vehicle with a reaction time lag. If initial speed difference and acceleration of following has an initial value, it will cause change in later space headway and amplifies through car-following model. Therefore, the initial speed difference and acceleration of following vehicles were set to 0 when they entered the simulation.

The initial space headway was driven with function (1) where at the entering time of following vehicle t_1:

$$\begin{cases} v_0(t_1) - v_1(t_1) = 0 \\ a_1(t_1) = 0 \end{cases} \tag{6}$$

Therefore, initial space headway and time headway are:

$$\begin{cases} s_1^*(t_1) = \delta + \tau \cdot v_1(t_1) \\ t_{hdw} = \dfrac{s_1^*(t_1)}{v_0(t_1)} \end{cases} \tag{7}$$

Apart from the initial entering state, vehicles will not immediately react to leading vehicle due to reaction time in car-following model. The accumulation of time lag in reaction may cause instable behaviors like collision and rapid acceleration in the platoon. To eliminate this effect, vehicles should react according to car-following model immediately after entering the sag section. The car-following model is set to function when the entering time of the leading vehicle t0 is bigger than the entering time of the following vehicle t1 minus reaction time Δt. The assumption is illustrated in Fig. 10).

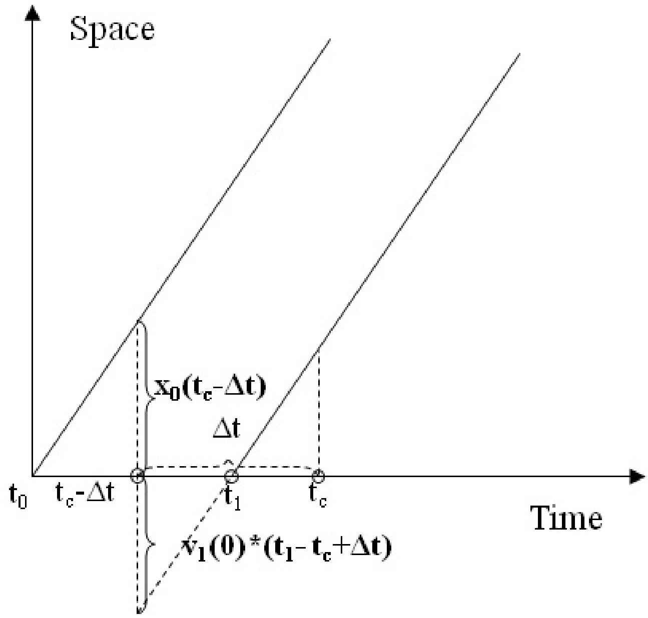


Fig. 10 Calculation of car-following behavior entering simulation

The simulation is stopped under 2 circumstances: 1. If the spacing between two adjacent vehicles is smaller than 4m, which indicates a rear end collision between vehicles; 2. If the speed is zero, which indicates a sudden stop of the vehicle. Collision and sudden stop are due to the linear construction of car-following model that the parameters combination may generate some very sensitive vehicles that accelerate and decelerate too rapidly when congestion is forming. In real world, this will not happen. Vehicles will adjust to the traffic condition much more smoothly. Therefore, collision and sudden stops are recognized as aggressive behavior and excluded from analysis.

Three kinds of leading car behavior are simulated for every single car-following parameter set where all the following cars in 5-min simulation have the same car-following parameter. There is no significant difference in congestion occurrence. Therefore the type 2. The non speed restore driver is used as a the leading car behavior in further research.

5-min simulation is carried out 1000 times with difference random seeds for every traffic demand level of 1250 veh/h/lane, 1500 veh/h/lane, 1750 veh/h/lane, 2000 veh/h/lane, 2250 veh/h/lane and 2500 veh/h/lane. The entering headway of vehicles obeys negative exponential distribution with λ = mean headway under certain traffic demand with a lower bound of 0.8s. The car-following parameters of following cars are chosen based on the desired spacing having minimum absolute difference with random entering headway. The congestion occurrence probability is calculated with the percentage of the cases of congestion in all the valid cases in 1000 simulations.

The congestion occurrence probability in simulation is then compared with the congestion occurrence probability observed by Oguchi (Oguchi, Katakura, & Shikada, 2001) at a different sag section in Japan (See Fig. 11).

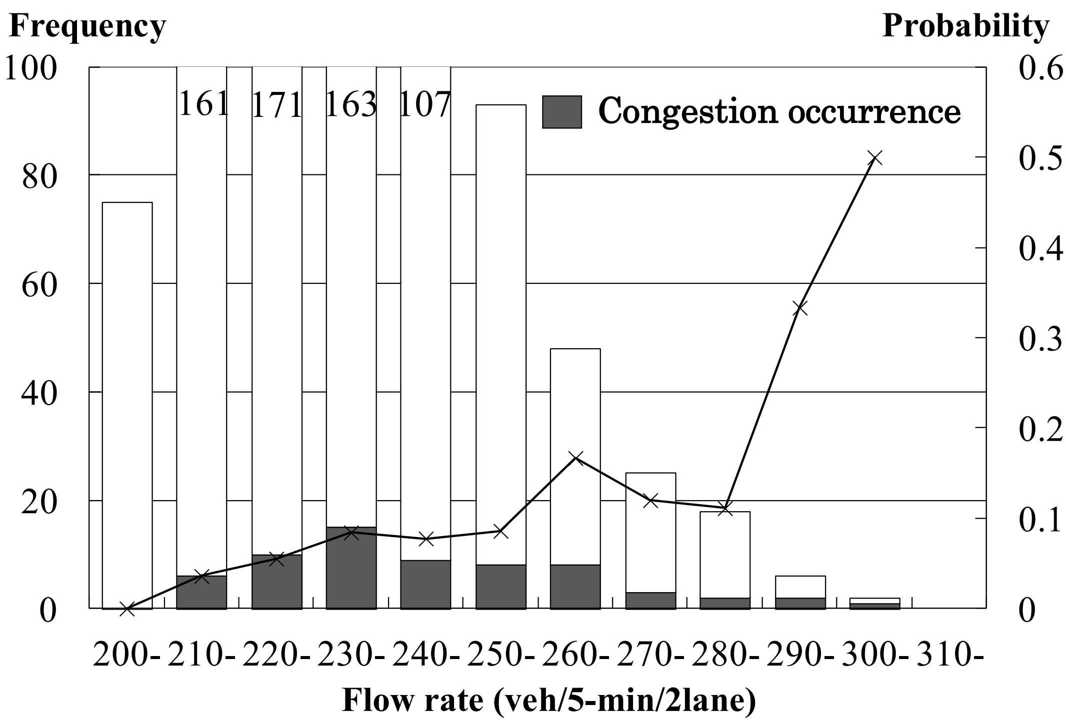


Fig. 11 Congestion occurrence observed on Japanese expressway bottleneck

To compare the congestion occurrence probability with field observation data where the traffic demand is calculated as veh/5-min/2lane, the lane use rate under different link flow rate is used to transfer the lane base flow rate in simulation study to the link base flow rate in field observation. The function and parameters are proposed by Hong (Hong, 2008).

$$L_u = \exp(a_2 Q) \tag{8}$$

where,

- Lu = Lane use rate on outer lane;
- Q = Link flow rate (veh/h);
- a2 = Constant , -3.541e-04 under speed limit of 100km/h;

Therefore, flow rate is calculated according to the following estimation function.

$$Q_i = Q[1-\exp(a_2 Q)]$$
(9)

where,

- Qi = Flow rate on (veh/h);

The tendency of congestion occurrence probability grows with traffic demand can be observed in both simulation with estimated car-following behaviors and field observation (See Fig. 12). Simulation with estimated car-following behavior can reproduce the congestion occurrence probability at a certain sag section. The congestion occurrence probability with car-following behaviors observed in congestion formation at Yamato sag section is higher than that of the field observation when traffic demand grows. The causes of this difference are different vertical slope condition at different sag section and different observation cases.

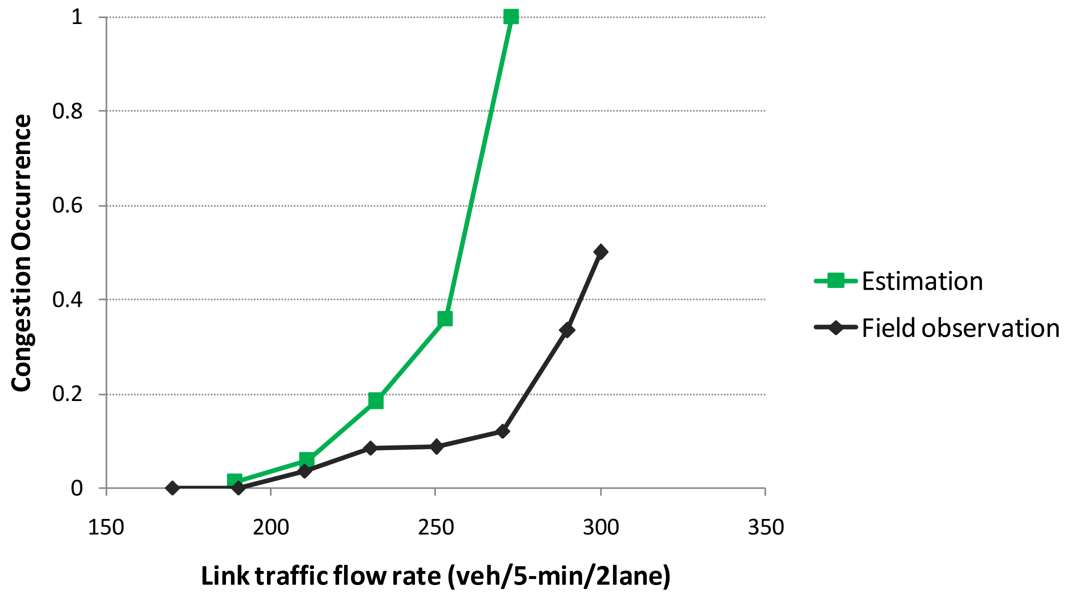


Fig. 12 Comparison of congestion occurrence with estimated car-following behavior and observation at another site

The calibration process has proved that the car-following model can well reproduce microscopic car-following behavior of most drivers as well as the macroscopic traffic condition occurring at sag section.

5. Results

Distribution of car-following parameters are studied as the variability of car-following behavior.

The correlation of parameters are firstly checked with Pearson's r to reveal the inner connection of car-following model parameters. The result is shown in Table 4.

Table 4 Correlation of car-following model parameters

	α_1	α_2	β	Δt	τ	δ
α_1	-					
α_2	0.2786^{**}	-				
β	-0.0231	-0.0448	-			
Δt	-0.4354^{**}	-0.1939^{**}	-0.0101	-		
τ	-0.2477^{**}	-0.1382^{**}	-0.1795^{**}	-0.0192	-	
δ	-0.0118	0.0720	0.0862^{*}	0.1720^{**}	-0.7380^{**}	-

$^{**}p<0.01$
$^{*}p<0.05$

Pearson's r of δ and τ is -0.7380 and significant. This indicates the desired spacing parameters δ and τ have high negative linear correlation. Therefore, they need to be estimated simultaneously in a joint distribution.

The joint cumulative frequency of δ and τ is shown in Fig. 4.7.6. It is consisted with a distribution of δ along the τ<0.2 range and a distribution of δ and τ at other value ranges. The former composition is a single distribution of δ and the latter composition is a joint distribution of δ and τ. The two parts are shown in Fig. 13 and Fig. 14.

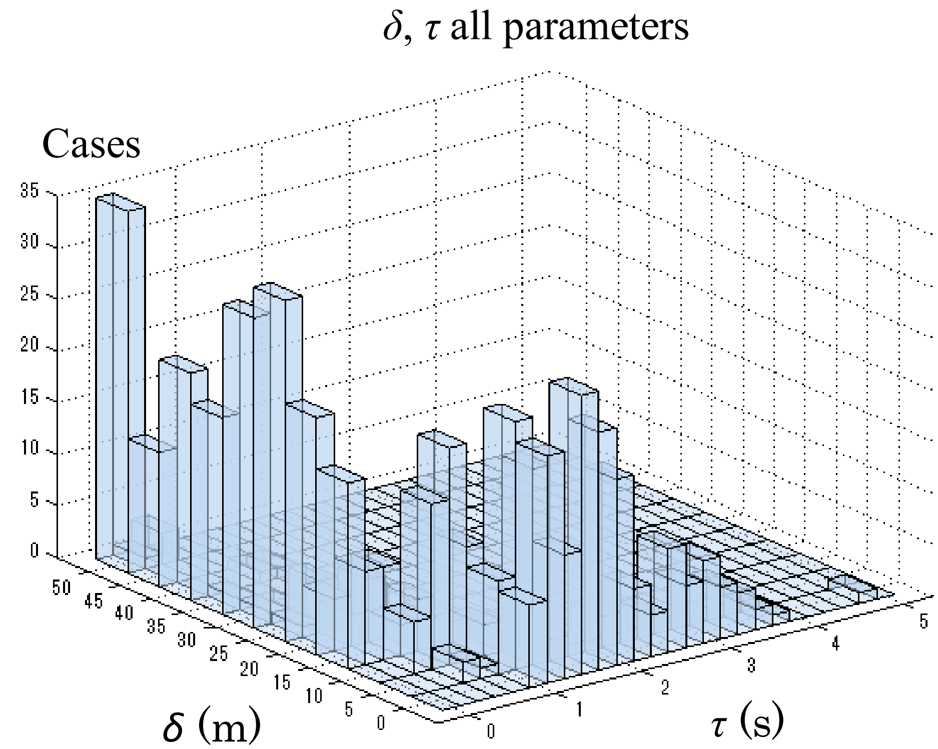


Fig. 13 Joint cumulative frequency of δ and τ

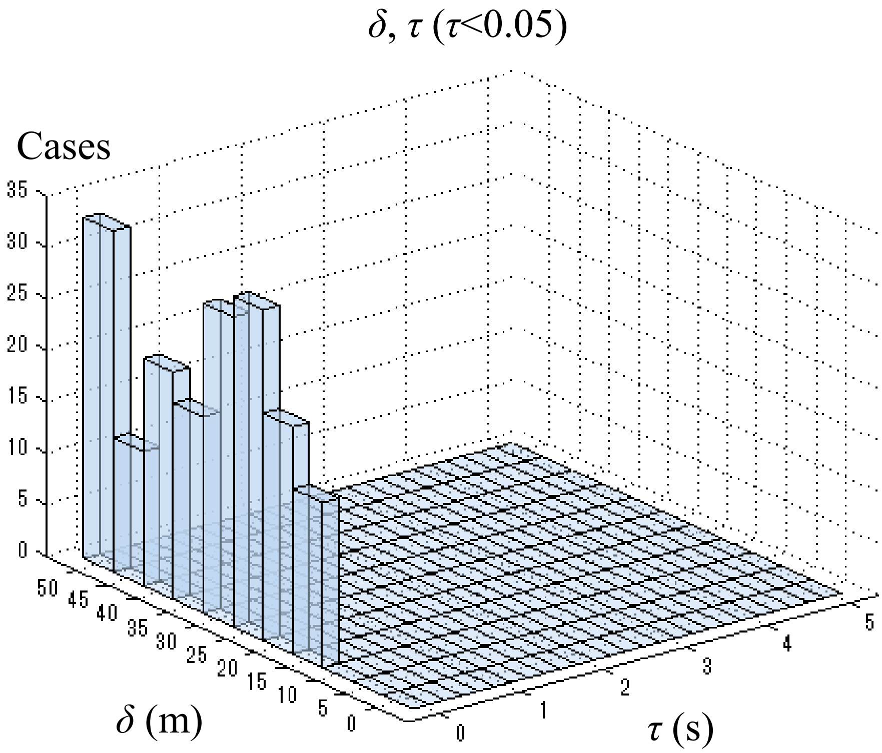


Fig. 14 1st part of the joint distribution of δ and τ

There are 187 cases in the first part of the joint distribution of δ and τ, which consists 34.3% of all the cases. The distribution of these cases is a shifted gamma distribution Gamma (1.8478,0.1019) with an intercept of 16m, which is the minimum spacing of vehicles in congestion. The estimated result is shown in Fig. 15.

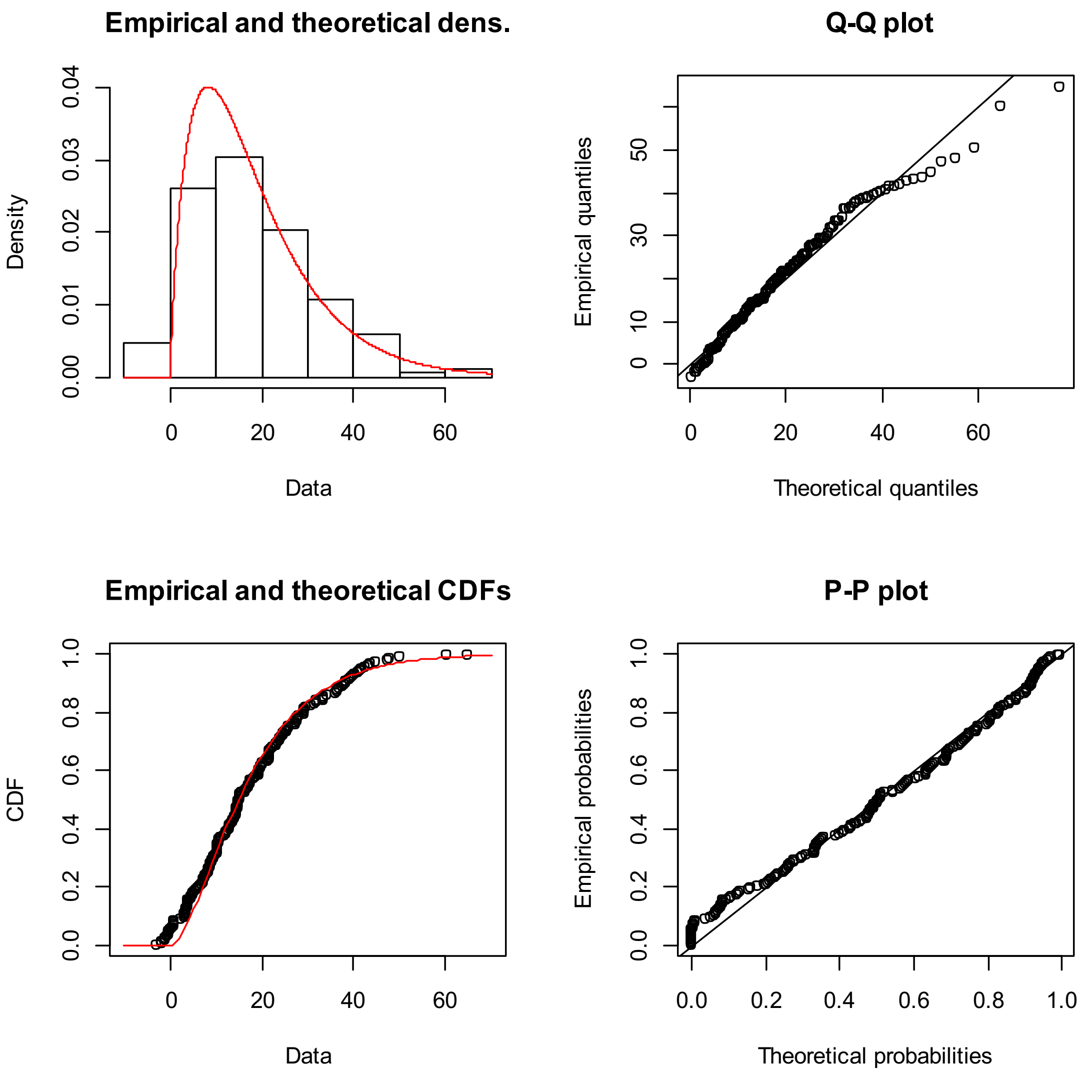


Fig. 15 Distribution estimation of the 1st part of the joint distribution of δ and τ

The rest 356 cases are in the second part of the joint distribution which is a bivariate gamma distribution.

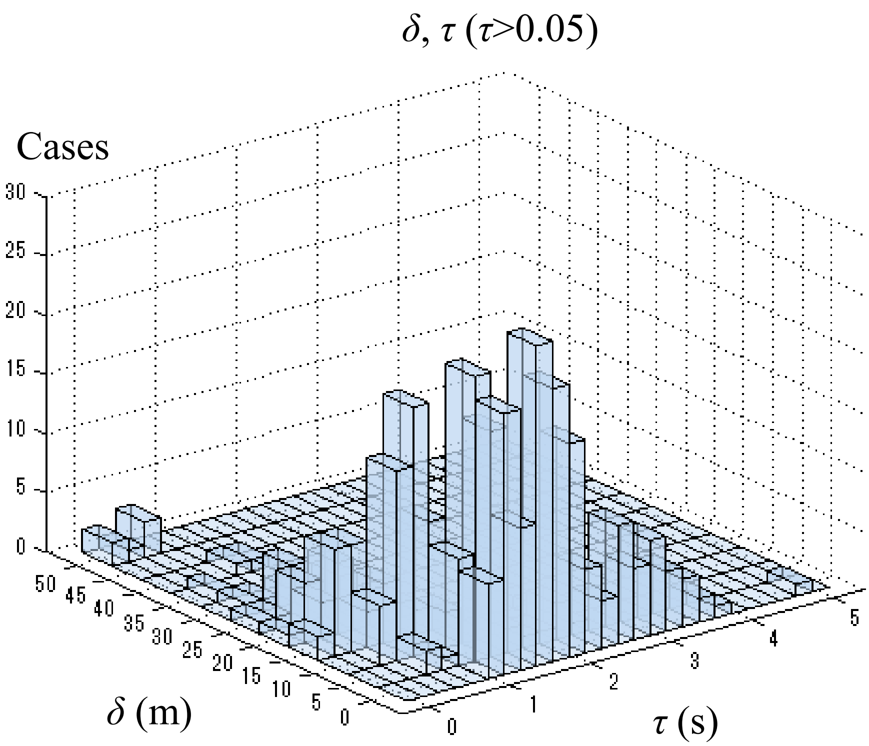


Fig. 16 2nd part of the joint distribution of δ and τ

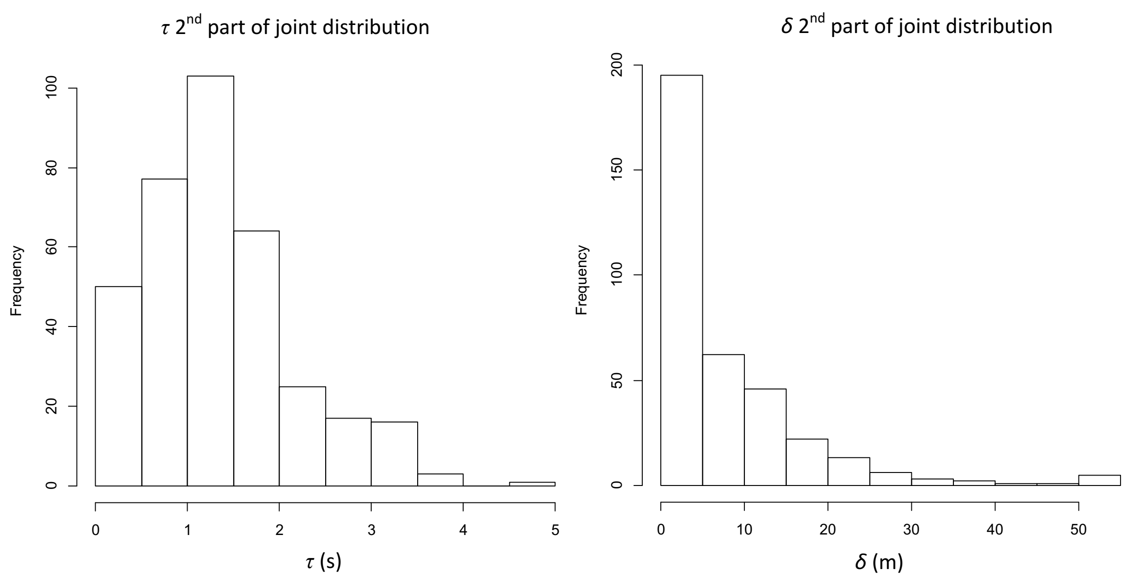


Fig. 17 Cumulative frequency of δ and τ of 2nd part of joint distribution

Most drivers have an adequate desired spacing consisting with desired headway around 1.2s and standstill spacing around 5m. $(5 - \tau)/5$ and δ obeys a bivariate Mckay's gamma distribution of p=0.4954, q=4.4171,a= 0.1476.

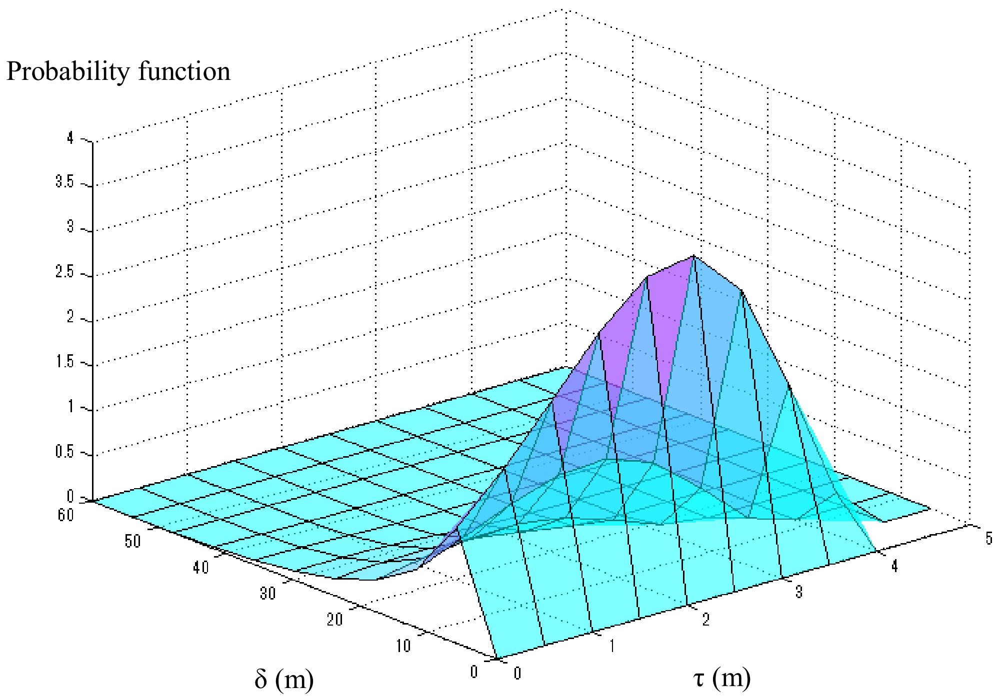


Fig. 18 Estimated joint distribution of δ and τ of 2nd part of joint distribution

Pearson's r of α_1 and Δt is -0.4354 and significant. It's not a very strong linear correlation. But Helly(Helly, 1959) suggested that negative correlation existed in α_1 and Δt. Therefore, they are further studied. Other correlations are small enough to be neglected. Fig. 19 shows the relationship between all α_1 and Δt. When Δt>1.1s, most α_1 is smaller than 0.5. on the other hand, when Δt ≤1.1s, α_1 is more evenly distributed within the value range.

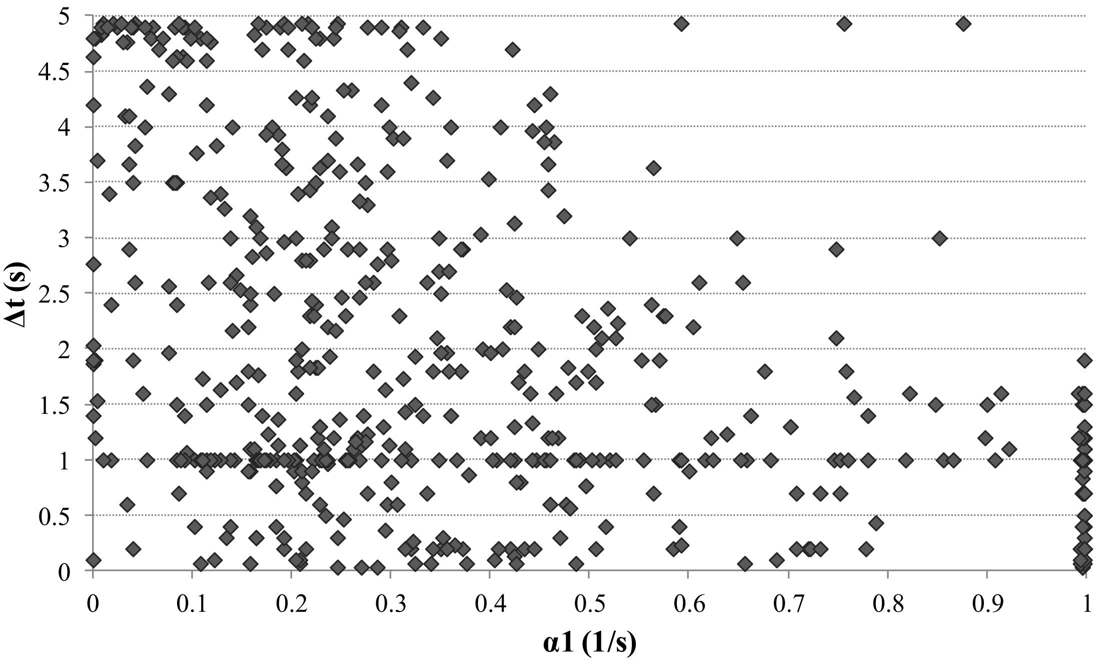


Fig. 19 Relationship between α_1 and Δt

The distribution of other parameters is separately estimated. As shown in Fig. 20, $\alpha 1$ is of a Gamma distribution: Gamma(1.689480,4.266007).

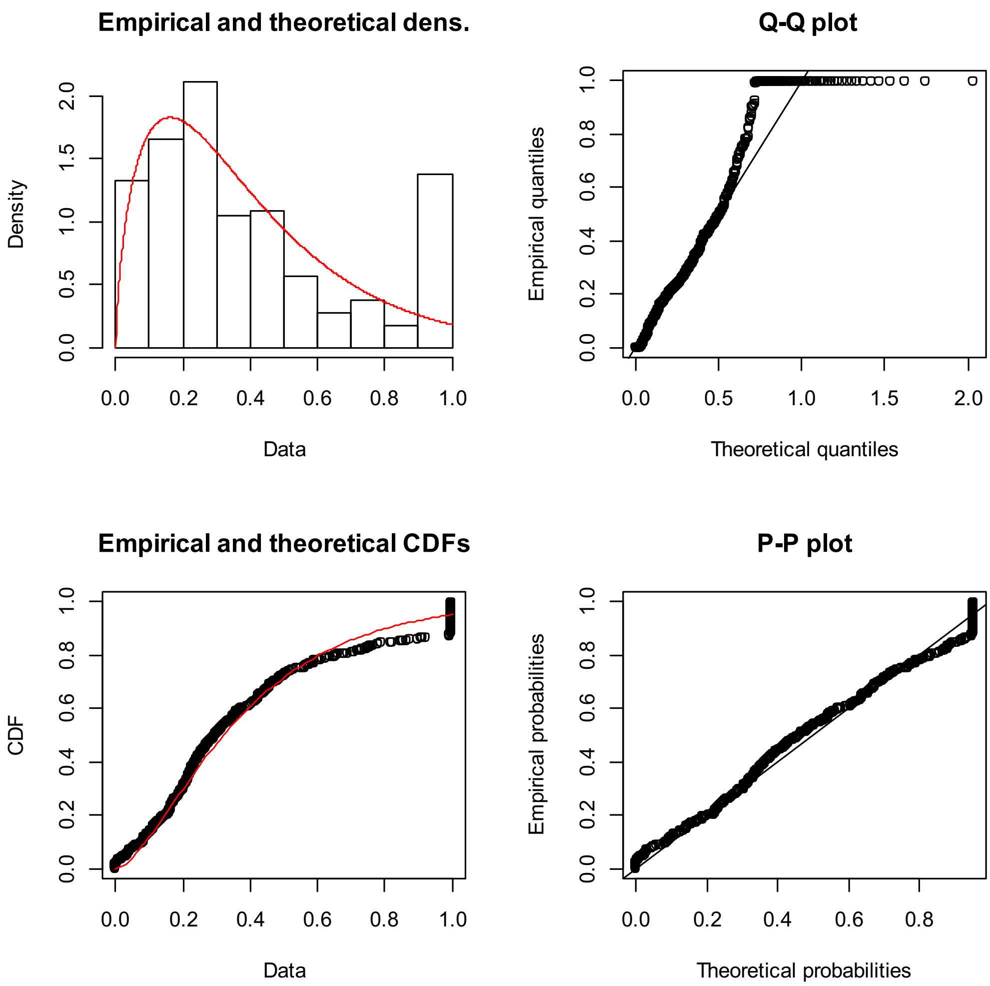


Fig. 20 Distribution of α_1

As shown in Fig. 21, $\alpha 2$ is of a negative exponential distribution: Exp(26.05566). Spacing with preceding vehicle has a minor effect on most drivers.

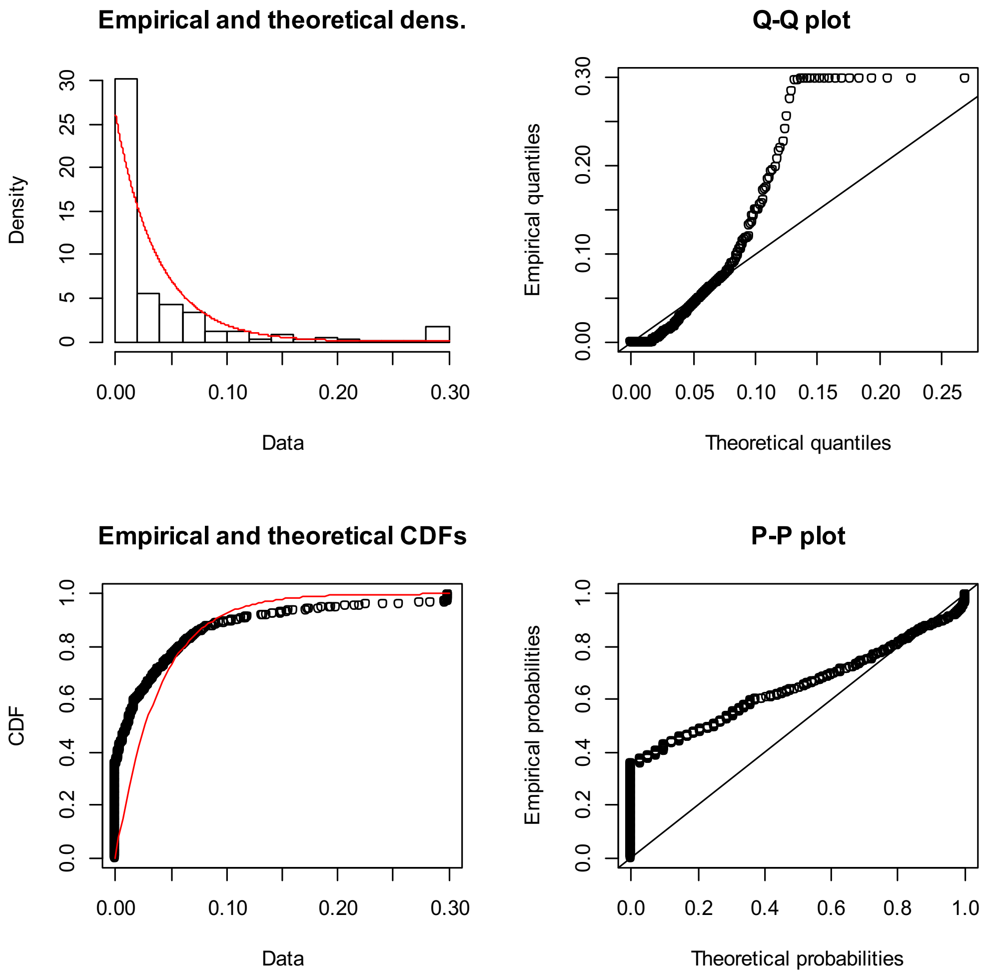


Fig. 21 Distribution of α_2

As shown in Fig. 22, β is of a beta exponential distribution: Beta (0.3568652, 0.2196074). The distribution of β has two peaks concentrating on the value range, which indicates that there are drivers who are not affected by the vertical grade while others are.

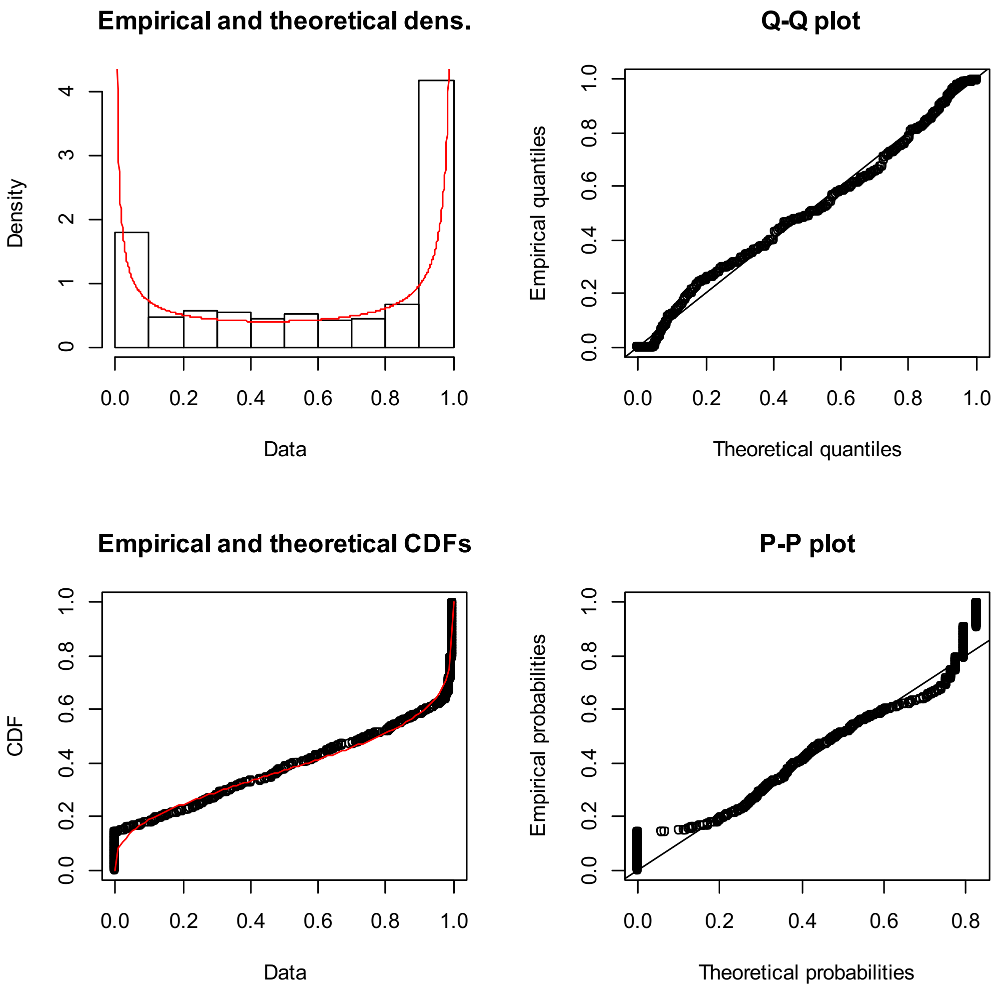


Fig. 22 Distribution of β

As shown in Fig. 23, Δt is of a Gamma exponential distribution: Gamma(1.7081664, 0.8675302). Most drivers have a reaction time around one second.

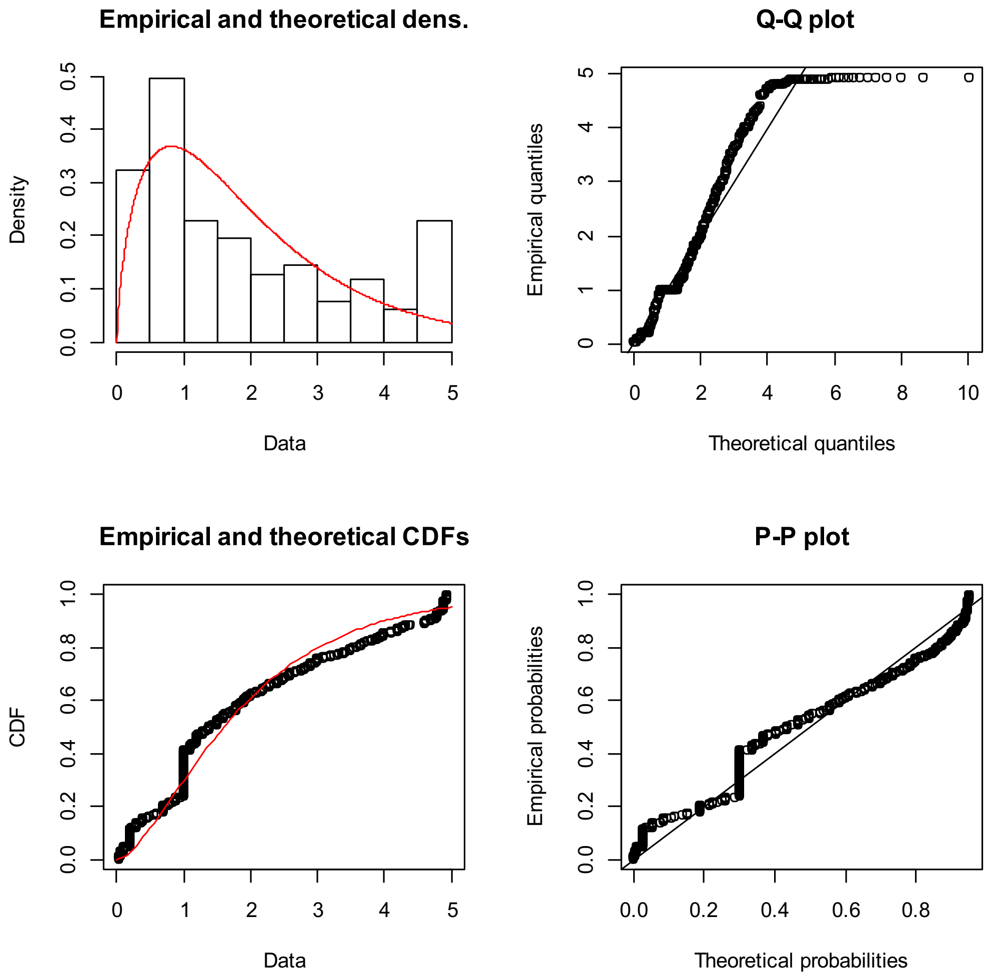


Fig. 23 Distribution of Δt

These distributions of parameters are utilized in simulation and the result is compared with that of the estimated parameters. The congestion occurrence probability is compared to the simulation result of actual estimated value of parameters (See Fig. 24). The congestion occurrence probability is higher than that of the estimated value under the 5 minutes traffic demand of 210-270 veh/2lane. There are two possible causes. One is that the proportion of different kinds of drivers may not be exactly random. The other is that there exists some potential relationship between certain parameters, for example, α_1 and Δt, etc.

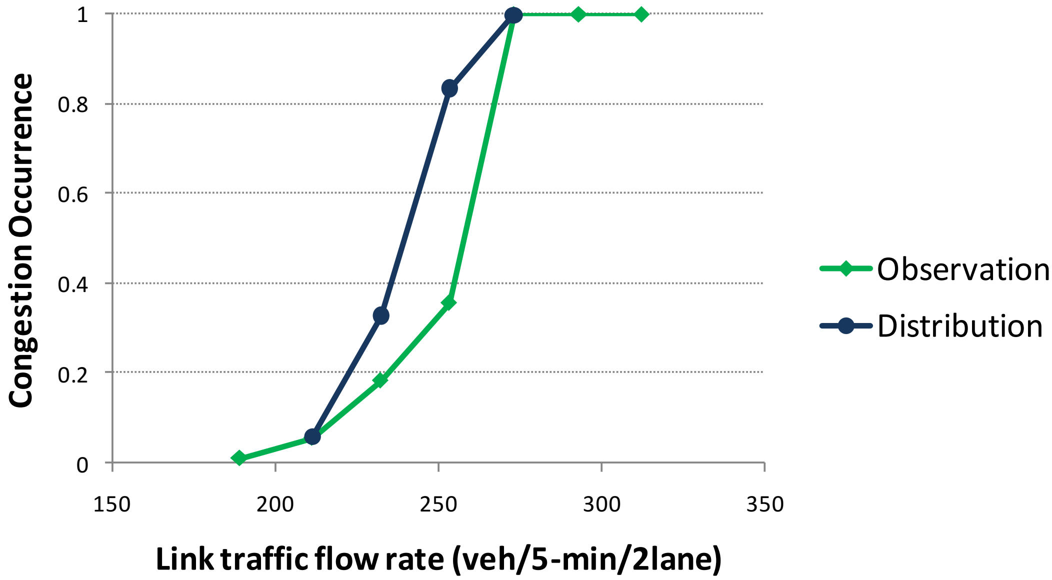


Fig. 24 Congestion occurrence probability of estimated parameters values and distribution

6. Conclusion and discussion

This research presents a simulation study of probabilistic congestion occurrence at expressway basic segment bottlenecks well known as sag section in Japan with individual variations in car-following behavior obtained from an actual sag section. A sag section is a road segment on an expressway in which the vertical slope increases at a small but constant rate. Through many previous researches, it is found impossible to predict congestion directly from detector data because the only observable flow condition changes happen right at the time congestion occurs. The propagation of unnoticed deceleration due to the vertical slope increase has long been researched as the cause of congestion. Perturbation amplifies in dense vehicle platoon consisted with drivers of varying car-following behavior. In this research, a car-following behavior model considering both the intra-driver variations and inter-driver variations is utilized and calibrated with field observation data. The variations are described with parameter distribution of car-following model. A simulation to reproduce traffic condition with the estimated parameters is thus proposed and validated in order to predict the probability of congestion occurrence at basic segment bottlenecks as well as test possible countermeasures to prevent and alleviate bottleneck congestion.

Two datasets containing all together 875 trajectories during congestion formation at the Yamato sag section are used in car-following behavior modeling. Trajectories of independent vehicles which do not react to the driving behavior of preceding vehicle and vehicles already involved in congestion are excluded from analysis. A car-following model considering both the intra-driver variations and inter-driver variations is utilized. The car-following model is based on a classic simple linear model which is found to be suitable for describing driving behavior during congestion formation at sag section. It is consisted of speed difference and spacing difference to its desired spacing with a time lag as well as an unnoticed deceleration caused by the vertical grade change at sag section. The inter-driver variability is described by car-following parameter values for every individual driver. The combination of direct estimation of observable parameters and heuristic search method for unobservable parameters are present for obtaining well-fitted and reasonable car-following model parameters distribution. Model calibration is made with objective function and performance indicators as well as simulation study. The objective function and performance indicators are studied to ensure that the model and its parameters can reproduce car-following behavior well. The simulation is carried out to reproduce the probabilistic nature of traffic congestion occurrence at a bottleneck of sag section including the effect of vertical slope increase. It successfully reproduces the tendency that the congestion occurrence probability grows

with traffic demand. A higher traffic demand will cause higher probability of congestion occurrence at sag section, which is the same in reality.

Finally, the variability of car-following behavior in drivers at sag section is studied with car-following parameter distribution. The relationships between different parameters are studied and no cross correlations excepted for the two parameters of desired speed are found. The desired spacing parameters are jointly distributed. The vertical effect parameter has two peaks concentrating on the value range, which indicates that there are drivers who are not affected by the vertical grade while others are. Congestion occurrence probability is higher in simulation with parameter values generated from distribution than in simulation with actual estimated value whose causes remain for future study. There are two possible causes. One is that the proportion of different kinds of drivers may not be exactly random. The other is that there exists some potential relationship between certain parameters, for example, α_1 and Δt, etc.

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