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A GENERAL COHERENCE RESULT

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Category Theory has a long history of coherence results. Herein, we state and prove a general coherence result that subsumes several of the results in the literature. This partially resolves a conjecture of Kelly in 1974, and it provides a simple general criterion for coherence that is satisfied by monadic instances of current interest. Several examples of particular interest are illustrated in detail.

1. Introduction

Over the past twenty-five years, Category Theory has seen an abundance of coherence results, commencing with MacLane's paper [8]. The introduction to [5], published in 1974, gives a general commentary on coherence problems. That commentary remains valid today, although the nomenclature has since evolved slightly.

Originally, results about certain diagrams commuting used to be seen as constituting the essence of a coherence theorem. In [5], Kelly argued that, although such results may be important consequences of a coherence theorem, they no longer constitute its essence. He outlined several approaches to coherence. In particular, the notion of 'club' had been developed specifically in order to study coherence problems [3]. Indeed, based upon his introduction to [5], Kelly had conjectured that for every 2-monad **T** on **Cat**, every pseudo-algebra is equivalent, in the appropriate 2-category of algebras, algebra maps, and algebra 2-cells, to a strict algebra. In [5], he argued that this is perhaps the ideal coherence result. Herein, it is answered affirmatively for a large class of 2-monads and groupoid-enriched monads, including all **Set**-based clubs, the 2-monad on $\mathbf{Cat}^{X\times X}$ whose algebras are 2-categories with object-set X, and, for any small 2-category $\mathbf{C}$, the 2-monad on $\mathbf{Cat}^{|\mathbf{C}|}$ whose algebras are 2-functors from $\mathbf{C}$ to **Cat**.

One limitation of this result should be mentioned. In [4, Section 3], Kelly defined

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a class of monads on **Cat** called the 'flexible' monads. These monads have a certain laxness in their algebras: for instance, they include the monad whose strict algebras are monoidal categories, but not the monad whose strict algebras are strict monoidal categories. For these flexible monads, it is easy to show that every pseudo-algebra is isomorphic to a strict algebra; so for instance, if **T** is the monad for monoidal categories, every pseudo-**T**-algebra is isomorphic to a monoidal category. Consequently, although the condition of our theorem is satisfied in this case, it adds nothing to our knowledge.

Section 2 revises the relevant notation; Section 3 gives the major result; and Section 4 gives a series of examples. Several of those examples will be used later in the series of articles on two-dimensional universal algebra currently being written by Kelly, other colleagues, and myself.

2. Notation

2.1. **Cat** is the 2-category of categories, functors, and natural transformations.

2.2. $\mathbf{Cat}_g$ is the sub-2-category of **Cat** given by categories, functors, and natural isomorphisms: one 'discards the non-invertible 2-cells' of **Cat**.

2.3. A 2-monad **T** on a 2-category **K** is a **Cat**-enriched monad, i.e. an endo-2-functor T with 2-natural transformations $\mu : T^2 \to T$ and $\tau : 1 \to T$ satisfying the usual three equations.

2.4. Given a 2-monad **T** on **K**, a (strict) **T**-algebra is precisely as for ordinary categories; a pseudo-**T**-algebra is given by $(A, a, \bar{a}, a^\circ)$, where $A \in \mathbf{K}$, $a : TA \to A$, and $\bar{a}, a^\circ$ are invertible 2-cells,

$$\begin{tikzcd} A \arrow[d, "\tau A"'] \arrow[dr, "1"] \arrow[dr, phantom, "\Downarrow a^\circ", pos=0.3, swap] & \\ TA \arrow[r, "a"'] & A \end{tikzcd} \qquad \begin{tikzcd} T^2A \arrow[r, "Ta"] \arrow[d, "\mu A"'] \arrow[dr, phantom, "\Downarrow \bar{a}"] & TA \arrow[d, "a"] \\ TA \arrow[r, "a"'] & A \end{tikzcd}$$

subject to three coherence axioms:

$$\begin{tikzcd} TA \arrow[d, "T\tau A"'] \arrow[dr, "1"] \arrow[dr, phantom, "\Downarrow Ta^\circ", pos=0.3, swap] & \\ T^2A \arrow[r, "Ta"'] \arrow[d, "\mu A"'] \arrow[dr, phantom, "\Downarrow \bar{a}"] & TA \arrow[d, "a"] \\ TA \arrow[r, "a"'] & A \end{tikzcd} \quad = \quad \text{identity} \tag{2.1}$$

$$
\begin{tikzcd}
TA \arrow[r, "a"] \arrow[d, "\tau TA"'] & A \arrow[d, "\tau A"] \arrow[ddr, "1"] & \\
T^2A \arrow[r, "Ta"] \arrow[dr, "\mu A"'] & TA \arrow[dr, "a"'] \arrow[d, phantom, "\Downarrow \bar{a}"] \arrow[ur, phantom, "\Downarrow a^\circ"] & \\
 & TA \arrow[r, "a"'] & A
\end{tikzcd}
\quad = \quad \text{identity} \tag{2.2}
$$

$$
\begin{tikzcd}
T^3A \arrow[r, "T^2a"] \arrow[d, "\mu TA"'] & T^2A \arrow[d, "\mu A"] \arrow[dr, "Ta"] & \\
T^2A \arrow[r, "Ta"] \arrow[dr, "\mu A"'] & TA \arrow[dr, "a"] \arrow[d, phantom, "\Downarrow \bar{a}"] \arrow[r, phantom, "\Downarrow \bar{a}"] & TA \arrow[d, "a"] \\
 & TA \arrow[r, "a"'] & A
\end{tikzcd}
=
\begin{tikzcd}
T^3A \arrow[r, "T^2a"] \arrow[d, "\mu TA"'] \arrow[dr, "T\mu A"] & T^2A \arrow[dr, "Ta"] \arrow[d, phantom, "\Downarrow T\bar{a}"] & \\
T^2A \arrow[dr, "\mu A"'] & T^2A \arrow[r, "Ta"] \arrow[d, "\mu A"] \arrow[dr, phantom, "\Downarrow \bar{a}"] & TA \arrow[d, "a"] \\
 & TA \arrow[r, "a"'] & A
\end{tikzcd}
\tag{2.3}
$$

2.5. A morphism of pseudo-**T**-algebras from $(A, a, \bar{a}, a^\circ)$ to $(B, b, \bar{b}, b^\circ)$ is given by the data

$$
\begin{tikzcd}
TA \arrow[r, "Tf"] \arrow[d, "a"'] \arrow[dr, phantom, "\Downarrow \bar{f}"] & TB \arrow[d, "b"] \\
A \arrow[r, "f"'] & B
\end{tikzcd}
$$

where $\bar{f}$ is an invertible 2-cell, subject to two coherence conditions:

$$
\begin{tikzcd}
A \arrow[r, "f"] \arrow[d, "\tau A"'] & B \arrow[d, "\tau B"] \arrow[ddr, "1"] & \\
TA \arrow[r, "Tf"] \arrow[dr, "a"'] & TB \arrow[dr, "b"'] \arrow[d, phantom, "\Downarrow \bar{f}"] \arrow[ur, phantom, "\Downarrow b^\circ"] & \\
 & A \arrow[r, "f"'] & B
\end{tikzcd}
=
\begin{tikzcd}
A \arrow[r, "f"] \arrow[d, "\tau A"'] \arrow[ddr, "1"] & B \arrow[ddr, "1"] & \\
TA \arrow[dr, "a"'] \arrow[r, phantom, "\Downarrow a^\circ"] & & \\
 & A \arrow[r, "f"'] & B
\end{tikzcd}
\tag{2.4}
$$

$$
\begin{tikzcd}
T^2A \arrow[r, "T^2f"] \arrow[d, "\mu A"'] \arrow[dr, "Ta"] & T^2B \arrow[dr, "Tb"] \arrow[d, phantom, "\Downarrow T\bar{f}"] & \\
TA \arrow[dr, "a"'] \arrow[r, phantom, "\Downarrow \bar{a}"] & TA \arrow[r, "Tf"] \arrow[d, "a"] \arrow[dr, phantom, "\Downarrow \bar{f}"] & TB \arrow[d, "b"] \\
 & A \arrow[r, "f"'] & B
\end{tikzcd}
=
\begin{tikzcd}
T^2A \arrow[r, "T^2f"] \arrow[d, "\mu A"'] & T^2B \arrow[d, "\mu B"] \arrow[dr, "Tb"] \arrow[r, phantom, "\Downarrow \bar{b}"] & \\
TA \arrow[r, "Tf"] \arrow[dr, "a"'] & TB \arrow[dr, "b"] \arrow[d, phantom, "\Downarrow \bar{f}"] & TB \arrow[d, "b"] \\
 & A \arrow[r, "f"'] & B
\end{tikzcd}
\tag{2.5}
$$

2.6. An algebra 2-cell $\alpha : (f, \bar{f}) \Rightarrow (g, \bar{g})$ is just a 2-cell $\alpha : f \Rightarrow g$ in **K** such that

$$
\begin{tikzcd}
TA \arrow[rr, bend left, "Tf"] \arrow[rr, bend right, "Tg"'] \arrow[rr, phantom, "\Downarrow T\alpha"] \arrow[d, "a"'] & & TB \arrow[d, "b"] \\
A \arrow[rr, bend right, "g"'] \arrow[rr, phantom, "\Downarrow \bar{g}"] & & B
\end{tikzcd}
=
\begin{tikzcd}
TA \arrow[rr, bend left, "Tf"] \arrow[rr, bend right, "f"'] \arrow[rr, phantom, "\Downarrow \bar{f}"] \arrow[d, "a"'] & & TB \arrow[d, "b"] \\
A \arrow[rr, bend right, "g"'] \arrow[rr, phantom, "\Downarrow \alpha"] & & B
\end{tikzcd}
\tag{2.6}
$$

2.7. **PS-T-Alg** is the 2-category of pseudo-**T**-algebras, morphisms, and algebra 2-cells, with the evident composition.

3. The main result

3.1. Definitions. (i) An equivalence in a 2-category is given by 1-cells $g : A \to B$, $f : B \to A$, and invertible 2-cells $\eta : 1 \Rightarrow gf$ and $\varepsilon : fg \Rightarrow 1$.

(ii) An equivalence is called an adjoint equivalence if η and ε are the unit and counit respectively of an adjunction.

It is straightforward to show that given any equivalence $(f, g, \eta, \varepsilon)$, replacing ε by $\varepsilon' = \varepsilon \cdot f\eta^{-1}g \cdot \varepsilon^{-1}fg$ gives an adjoint equivalence.

3.2. Theorem. (Kelly [4]). *If* **T** *is a 2-monad on* **K**, *if* $(A, a, \bar{a}, a^\circ)$ *and* $(B, b, \bar{b}, b^\circ)$ *are pseudo-algebras,* $(f, \bar{f}) : A \to B$ *is an algebra map, and* $f \dashv g$ *is an adjoint equivalence in* **K**, *then the adjunction lifts to an adjoint equivalence in* **Ps-T-Alg**. □

This result is not stated explicitly in [4], which assumes only that $f \dashv g$ and concludes that g lifts to what we now call a lax morphism of algebras. Moreover, the result is only stated for strict algebras. However, the modifications needed for the pseudo-algebra case are trivial, and inspection of the proof reveals that if $f \dashv g$ is an adjoint equivalence, then the $\bar{g}$ constructed is an isomorphism, and we do have the desired lifting.

The following lemma, asserting that functors bijective on objects and fully-faithful functors form a factorization system on **Cat** with a two-dimensional property, is probably folklore:

3.3 Lemma. (i) *Every functor* $f : A \to B$ *in* **Cat** *can be factored as* gh, *where* g *is fully-faithful and* h *is bijective on objects.*

(ii) *Given a diagram in* **Cat** *of the form*

$$
\begin{tikzcd}
A \arrow[r, "h"] \arrow[d, "u"'] \arrow[dr, phantom, "\Downarrow \alpha"] & B \arrow[d, "v"] \\
C \arrow[r, "g"'] & D
\end{tikzcd}
$$

where α is an invertible 2-cell, h is bijective on objects, and g is fully-faithful, there exist unique $w : B \to C$ and $\beta : v \Rightarrow gw$ such that $wh = u$ and $\beta h = \alpha$; β is necessarily iso.

Proof. (i) Well known.

(ii) Since h is bijective on objects and we must have $wh = u$, the object function of w must be $\mathbf{ob\,B} \xrightarrow{(\mathrm{ob}\,h)^{-1}} \mathbf{ob\,A} \xrightarrow{\mathrm{ob}\,u} \mathbf{ob\,C}$. Moreover, the data for β are completely determined by α, again since h is bijective on objects. In order to make β natural, w must be defined on homs by

$$\begin{aligned}\mathbf{B}(b,b') \xrightarrow{v} \mathbf{D}(vb,vb') = \mathbf{D}(vhh^{-1}b, vhh^{-1}b') &\xrightarrow{\alpha^{-1}\cdot(\ )\cdot\alpha} \mathbf{D}(guh^{-1}b, guh^{-1}b')\\ = \mathbf{D}(gwb, gwb'),&\end{aligned}$$

and using the fact that g is fully-faithful.

It is clear that this defines a functor, that β is natural and invertible, and that the two equations hold. By construction, w and β are unique. $\square$

3.4. Theorem. *Let* $\mathbf{T}$ *be a 2-monad on* $\mathbf{Cat}_g^X$, *where X is a small set. Suppose that T preserves 'bijections on objects', i.e. if $f_x : A_x \to B_x$ is a bijection on objects for all x, then $Tf_x : TA_x \to TB_x$ is a bijection on objects for all x. Then, every pseudo-*$\mathbf{T}$*-algebra is equivalent in* **Ps-T-Alg** *to a strict algebra.*

Proof. Given $(A, a, \bar{a}, a^\circ)$, it follows from Lemma 3.3(i) that there exist g and h with $a = gh$, g_x fully faithful, and h_x bijective on objects for each x. Indeed, g is an equivalence since $a^\circ : 1 \cong g \cdot h \cdot \tau A$, since $a^\circ g : g \cdot (h \cdot \tau A \cdot g) \cong g \cdot 1$, and since g_x is fully faithful for each x. By 3.1, we may assume that g is part of an adjoint equivalence. Let cod $h = B$. By Theorem 3.2, it suffices to give a strict algebra structure on B, and to show that g lifts to a morphism of algebras. Applying Lemma 3.3(ii) to

$$\begin{tikzcd} T^2A \arrow[r, "Th"] \arrow[d, "\mu A"'] & TB \arrow[r, "Tg"] \arrow[d, phantom, "\Downarrow\bar{a}"] & TA \arrow[d, "a"] \\ TA \arrow[r, "h"'] & B \arrow[r, "g"'] & A \end{tikzcd}$$

gives $b : TB \to B$ and $\bar{g} : a \cdot Tg \Rightarrow g \cdot b$, with $b \cdot Th = h \cdot \mu A$ and $\bar{g} \cdot Th = \bar{a}$. For the two coherence conditions, apply the unicity clause of Lemma 3.3(ii) to the following cases of (2.2) and (2.3):

$$\begin{tikzcd} TA \arrow[rr, "h"] \arrow[d, "\tau TA"'] & & B \arrow[rr, "g"] \arrow[d, "\tau B"'] & & A \arrow[d, "\tau A"'] \arrow[ddr, "1"] & \\ T^2A \arrow[rr, "Th"] \arrow[dr, "\mu A"'] & & TB \arrow[rr, "Tg"] \arrow[dr, "b"'] \arrow[rr, phantom, shift right=6, "\Downarrow\bar{g}"] & & TA \arrow[dr, "a"'] \arrow[r, phantom, "\Downarrow a^\circ"] & \\ & TA \arrow[rr, "h"'] & & B \arrow[rr, "g"'] & & A \end{tikzcd} \quad =$$

$$
= \begin{tikzcd}
TA \arrow[rr, "h"] \arrow[d, "\tau TA"'] \arrow[ddr, "1"] & & B \arrow[rr, "g"] \arrow[ddr, "1"] & & A \arrow[ddr, "1"] & \\
T^2A \arrow[dr, "\mu A"'] & & & & & \\
& TA \arrow[rr, "h"'] & & B \arrow[rr, "g"'] & & A
\end{tikzcd}
$$

for each x, and

$$
\begin{tikzcd}
T^3A \arrow[rr, "T^2h"] \arrow[dd, "\mu TA"'] & & T^2B \arrow[rr, "T^2g"] \arrow[dd, "\mu B"'] & & T^2A \arrow[dr, "Ta"] \arrow[dd, "\mu A"'] & \\
& & & & & TA \arrow[dd, "a"] \\
T^2A \arrow[rr, "Th"] \arrow[dr, "\mu A"'] & & TB \arrow[rr, "Tg"] \arrow[dr, "b"'] & & TA \arrow[dr, "a"'] \arrow[r, phantom, "\Downarrow\bar{a}"] & \\
& TA \arrow[rr, "h"'] & & B \arrow[rr, "g"'] \arrow[u, phantom, "\Downarrow\bar{g}"] & & A
\end{tikzcd}
$$

$$
= \begin{tikzcd}
T^3A \arrow[r, "T^2h"] \arrow[dr, "T\mu A"] \arrow[dd] & T^2B \arrow[r, "T^2g"] \arrow[dr, "Tb"'] & T^2A \arrow[dr, "Ta"] \arrow[d, phantom, "\Downarrow T\bar{g}"] & \\
& T^2A \arrow[r, "Th"] \arrow[dd, "\mu A"] & TB \arrow[r, "Tg"] \arrow[dd, "b"] \arrow[ddr, phantom, "\Downarrow\bar{g}"] & TA \arrow[dd, "a"] \\
T^2A \arrow[dr, "\mu A"'] & & & \\
& TA \arrow[r, "h"'] & B \arrow[r, "g"'] & A
\end{tikzcd}
$$

for each x.

These show that (B, b) is a strict algebra and that $(g, \bar{g})$ is a morphism of algebras, thus completing the proof. □

3.5. Corollary. *The analogous result holds for 2-monads* $\mathbf{T}$ *on* $\mathbf{Cat}^X$.

Proof. $\mathbf{T}$ restricts to a 2-monad $\mathbf{T}_g$ on $\mathbf{Cat}_g^X$; the algebras, pseudo-algebras and morphisms remain the same, and an equivalence of pseudo-$\mathbf{T}_g$-algebras is automatically an equivalence of pseudo-$\mathbf{T}$-algebras. The result now follows from Theorem 3.4. □

It is straightforward to generalise this result to functor 2-categories $[\mathbf{C}, \mathbf{Cat}]$ for small $\mathbf{C}$. However, it requires a succession of pasting diagrams, and it is not the case of primary interest; so I omit the proof.

4. Examples

4.1. Clubs. Those 2-monads on **Cat**, or on $\mathbf{Cat}^X$, described by **Set**- and $\mathbf{Set}^{\mathrm{op}}$-based clubs are discussed in detail in [3, Section 10], in which a description is given of the free algebras for the club. It is clear from this description that the condition of Theorem 3.4 holds, the objects of TA being formed purely from the objects of A. The crucial facts are summarized in [3, Section 10.2], which also explains why **Cat**-based clubs do not satisfy the condition. Some examples are the monads whose strict algebras are monoidal categories, categories with strictly associative finite products or coproducts, or strict symmetric monoidal categories.

The same is true for the mixed variance clubs of [2], such as the monad for strict symmetric monoidal closed categories. Recall from [1, Section 6] that these monads are defined only on $\mathbf{Cat}_g^X$, not on $\mathbf{Cat}^X$. A more general notion of 'club' is given in [6]. Using this more general notion, it follows that if S preserves 'bijective on objects' functors, then any club over $S1$ does likewise, so Theorem 3.4 applies. Our theorem also applies to the 'unclubbable' monads of [1, Section 4.3], such as that for Cartesian closed categories, for which the free algebras can still be given and which still preserve bijections on objects.

4.2. [C, Cat]. Let **C** be a small 2-category. Then, the functor 2-category $[\mathbf{C}, \mathbf{Cat}]$ is 2-monadic over $[|\mathbf{C}|, \mathbf{Cat}]$, the monad being given by left Kan extension: if **T** is the monad, $H \in [|\mathbf{C}|, \mathbf{Cat}]$, and $c \in \mathbf{C}$, then

$$(TH)c = \Sigma_{d \in |\mathbf{C}|} \mathbf{C}(d,c) \times Hd.$$

It is obvious that **T** satisfies the condition of Theorem 3.4. A pseudo-algebra $(H, h, \bar{h}, h^\circ)$ is given by $H \in [|\mathbf{C}|, \mathbf{Cat}]$, together with functors $h_c : \Sigma_d \mathbf{C}(d,c) \times Hd \to Hc$ and isomorphisms

$$\begin{tikzcd}
\Sigma_c \Sigma_d \mathbf{C}(e,d) \times \mathbf{C}(d,c) \times He \arrow[r, "\Sigma_d \mathbf{C}(d,c) \times h_d"] \arrow[d, "\Sigma_c \mu_{c,d,e} \times Hc"'] \arrow[dr, phantom, "\Downarrow \bar{h}_c"] & \Sigma_d \mathbf{C}(d,c) \times Hd \arrow[d, "h_c"] \\
\Sigma_c \mathbf{C}(e,c) \times He \arrow[r, "h_c"'] & Hc
\end{tikzcd}$$

and

$$\begin{tikzcd}
Hc \arrow[d, "\binom{j_c}{1_{Hc}}"'] \arrow[dr, "1"] \arrow[dr, phantom, "\Downarrow h_c^\circ" near start] & \\
\Sigma_d \mathbf{C}(d,c) \times Hd \arrow[r] & Hc
\end{tikzcd}$$

subject to three coherence axioms. This amounts precisely to a homomorphism from **C** to **Cat**. A morphism of pseudo-algebras $(\alpha, \bar{\alpha}) : (H, h, \bar{h}, h^\circ) \to (K, k, \bar{k}, k^\circ)$ is given by functors $\alpha_c : Hc \to Kc$, and isomorphisms

$$\begin{tikzcd}
\Sigma_d \mathbf{C}(d,c) \times Hd \arrow[rr, "\Sigma_d \mathbf{C}(d,c)\times\alpha_d"] \arrow[d, "h_c"'] \arrow[drr, phantom, "\Downarrow\bar{\alpha}_c"] & & \Sigma_d \mathbf{C}(d,c) \times Kd \arrow[d, "k_c"] \\
Hc \arrow[rr, "\alpha_c"'] & & Kc
\end{tikzcd}$$

subject to two coherence conditions.

This amounts precisely to a pseudo-natural transformation. An algebra 2-cell amounts to a modification. So, Theorem 3.4 shows that every homomorphism from **C** to **Cat** is equivalent, in **Hom(C, Cat)**, to a 2-functor.

4.3. $\mathbf{2\text{-}Cat}_X$. Given a small set X, let $\mathbf{2\text{-}Cat}_X$ be the 2-category given as follows: 0-cells are 2-categories with object-set X; 1-cells are 2-functors that are the identity on objects; 2-cells are lax-natural transformations that send objects to identity 1-cells.

The 2-functor $U: \mathbf{2\text{-}Cat}_X \to \mathbf{Cat}^{X\times X}$ sending $\mathbf{C}$ to $(\mathbf{C}(x,y) \mid x,y \in X)$ is 2-monadic: $\mathbf{2\text{-}Cat}_X$ is given by operations and equations on $\mathbf{Cat}^{X\times X}$. The 2-monad $\mathbf{T}$ can be constructed explicitly, and so can its pseudo-algebras, etcetera. In fact, this is just a simple generalization of the case of strict monoidal categories. Given $F: X\times X \to$ **Cat**, regard F as a 2-graph on X. Then, TF is the free 2-category on F, subject to the equations on 2-cells given by the category structure on each $F_{x,y}$. It is clear that $\mathbf{T}$ satisfies the condition of Theorem 3.4. The pseudo-algebras have an n-fold composition $\otimes_n$ for each $n \in \mathbf{N}$, subject to coherent isomorphisms $\otimes_n(\otimes_{m_1}, \dots, \otimes_{m_n}) \cong \otimes_{m_1+\dots+m_n}$. It follows just as in the strict monoidal case in [7], that **Ps-T-Alg** is equivalent to the 2-category of bicategories on X, homomorphisms that are the identity on objects, and lax-natural transformations that send objects to identity maps. It follows that every bicategory is biequivalent to a 2-category with the same set of objects.

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